Lecture Notes

for
EAS 201/ EAS203
The Physics and Chemistry of the Earth

How the Earth Works

by

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2004

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I. INTRODUCTION

Chapter 1: A Quick First Look – The Context

Look at the Earth from the perspective of an alien visitor to our solar system. The outer planets are gassy giants, nearly Suns themselves. The hard-surfaced moons of these planets are intensely cratered. Some of the moons have deformable (Ganymede) or tectonically active surfaces (Io) and are able to erase the evidence of past impacts, but others, such as the moon, record a history of bombardment over more than 4000 million years (4Ga).

The inner hard-surfaced planets are more like the moons of the outer gassy giants than the gassy giants themselves. The differences between the three planets within habitable distance from the Sun are striking. Mars is cold (polar caps of dry ice) and windy. Venus looks attractive but is much too hot. Its runaway greenhouse effect has raised its surface temperature over 400K to 737K (864F), well above the melting point of lead (500K). Most of the Earth has just the right temperatures to support life; it is a beautiful planet largely covered by oceans. Although some evidence of meteorite impact can be found, such as Meteor Crater, in Arizona and Manacougan Crater, in Quebec, the Earth shows remarkably few obvious signs of meteorite bombardment. This is because tectonic processes constantly produce new surface and erosion erases or mutes what evidence survives. The odd and very systematic variations of surface elevation in the oceans and continents are immediately noted. Also night-lights show the planet is densely populated by an intelligent species that is utilizing the resources of the planet in a somewhat wasteful fashion (slash and burn fires in the Amazon and Africa, the flaring of gas in Nigeria and the Arabian Peninsula).

Chapter 2: A Closer Look - Some Energetics

The masses of the planets and observations of meteorites and comets reveals a great deal about how the Earth was born. The mass of the planets is indicated by the revolution periods and mean orbital radii of their moons, both of which can be observed from Earth (Kepler's Laws, see Page 4). The densities of the inner planets are ~5 ton/m^3 (or, equivalently, Mg/m^3 or g/cc). The densities of the outer planets are ~1.3 ton/m^3. The outer surface of the Earth has a density of ~2.7 to 3 ton/m^3, which even if compressed by the pressures in the Earth's interior, could not account for the average 5.52 ton/m^3 density of the Earth. This indicates that the Earth is a chemically differentiated planet.

Comets are largely composed of frozen gas, including a lot of ice of density ~1 ton/m^3. Stony meteorites have a density of 3.5 ton/m^3. Iron meteorites have densities of ~7.5 ton/m^3. Thus, based on density considerations alone, the inner planets seem to be composed of a mix of stony and iron meteorite material, and the outer planets largely of cometary gases.

Closer examination of meteorites provides more information. There are three kinds of stony meteorites: chondrites, carbonaceous chondrites, and achondrites. Chondrites are formed from condrules, ~1 mm spherules that were once molten. The chondrules agglomerated with other material in space. Some of the chondrites contain carbonaceous material mixed in with the chondrules. Some carbonaceous chondrites contain material that could never have been heated above ~373K (100°C). From the absorption band intensities in the Sun's radiation spectrum, we know that the ratios of elements in carbonaceous chondrites are practically identical (within measurement error) to the ratios of elements in the Sun. The recent Galileo probe's measurements of Jupiter's outer atmosphere also found a gas composition similar to that of the sun. For these reasons it is thought that the Sun, planets, and carbonaceous chondrites accreted from the same material.

Initially there was a lot of cometary gas associated with the carbonaceous chondrite material that accreted to form the planets and Sun. As the Sun heated up, the solar wind blew the gas from the accreting inner planets. The inner planets have a residual chondrite composition and the outer planets contain chondrite and gas material in proportions more similar to that in the Sun.

The mix of elements in the planets and Sun tells us that the elements were synthesized by fusion reactions (to produce elements up to iron) and then by the intense neutron bombardment that occurs in supernova explosions (to synthesize elements heavier than iron). It is likely that agglomeration of gas and debris to form our solar system was triggered by a nearby supernova explosion. Short-lived radioactive elements produced in the supernova explosion like ^{26}Al helped the Earth heat up rapidly. Once heated sufficiently, the iron core of the Earth segregated, an event that could have released enough gravitational energy to heat the Earth to over 6000K – just by itself.

The Earth is believed to have been born of at least three gigantic explosions. (1) The original Big Bang that produced Hydrogen (H) and Helium (He) which collected in stars to synthesize elements up to iron, (2) supernova explosions whose intense neutron flux synthesized elements heavier than iron, (3) a local supernova that triggered the formation of our solar system. The Earth was formed late in the history of the universe. If it had not been late in formation, there would not have been time for elements heavier than iron to be synthesized in supernova explosions and the debris incorporated into our solar system.

The early Earth was intensely bombarded by meteorites. Analysis of the overlapping impact craters on the moon coupled with the known age's of parts of the moon's surface from radioactive age-dating of samples brought back by the Apollo and Luna missions indicates that the intensity of this bombardment dropped rapidly with time as the solar system aged.

In the Problem Set 2a you will calculate the relative magnitude of some important Earth events. You will find that the energy released by segregation of the central iron core is equivalent to the energy in 8×10^6 years of Sunlight falling on the Earth. Collision of the Earth with a 20 km diameter meteorite is equivalent to about 2.5 months of Sunlight, and a very large earthquake is equivalent to about 2 minutes of Sunlight.

Because the input of solar energy at a planet's surface is so much larger than the internal heat released from the planet's interior, the balance between incoming solar energy and the reflected and radiated thermal energy determines its surface temperature. If Venus, Earth and Mars had the black body (a theoretically ideal absorber and radiator of energy at all electromagnetic wavelengths, where all impinging radiation is emitted back at all wavelengths) temperatures required to radiate the absorbed incident electromagnetic energy they receive from the Sun, they would have temperatures of 328K, 278K, and 228K (55, 5 and -50°C) respectively. If account is taken of the fraction of sunlight reflected from each planet (their albedo), the black body temperatures of Venus, Earth and Mars would be 244K, 253K, and 213K (-29, -20 and -60°C) respectively The runaway greenhouse heating of Venus adds ~400K to its surface temperature and accounts for the ~730K (460°C) temperature of that planet. A 35°C greenhouse heating of the Earth gives it a habitable 15°C average temperature. For Mars, the greenhouse heating of 15K is insufficient to make it very comfortable (average T of 230K (-45°C)). Clearly we are dependent on greenhouse warming for our comfortable Earthly habitat. In fact, life may have something to do with regulating the CO_2 - greenhouse effect to keep Earth temperatures in a habitable range. The incineration of our neighbor Venus indicates we are right to be concerned, but we must be careful not to eliminate the greenhouse effect entirely, or we will freeze.

Additional Reading

W. S. Broecker, 1985, How to Build a Habitable Planet, Eldigio Press, LDEO, Box 2, Palisades, New York, 291 p.

2.1 The Mass and Density of Planets from Kepler's Laws

One of the most revealing observations that can be made of our solar system is the large difference in density between the inner terrestrial planets, and the outer gassy planets. We can measure a planet's density if its has a satellite orbiting it by using Kepler's Laws, since these relations reflect the balance between gravitational and centrifugal forces that determines the orbit of a planet or moon.

Kepler's Laws are:

(1) All planets move in elliptical orbits with the Sun at one focus. The other focus is located symmetrically at the opposite end of the elliptical orbit (Figure 2.1).

(2) A line joining any planet to the Sun sweeps out equal areas in equal times. As seen in Figure 2.1, if $\Delta t_1 = \Delta t_2$, then Area 1 = Area 2.

(3) The square of the planets orbital period is proportional to the cube of the planet's mean distance from the Sun ($T^2 = kr^3$, where k = constant of proportionality).

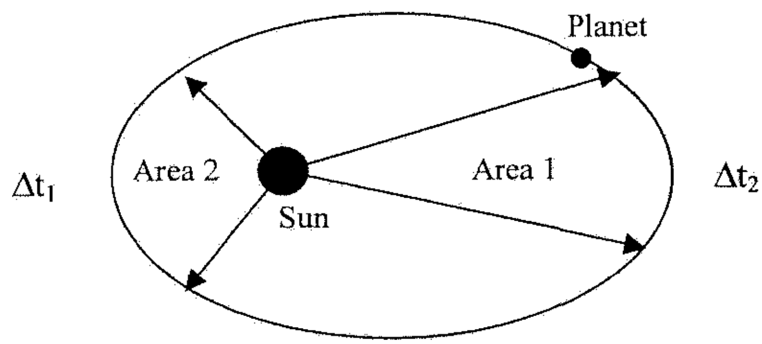


Figure 2.1 Kepler's Laws.

Kepler's Laws can be derived by balancing gravitational and centrifugal forces. If M is the mass of the Sun, m is the mass of a planet in orbit around the Sun, r is the mean orbital radius of that planet, G is the gravitational constant, and ω is the angular velocity of the planet ($\omega = 2\pi/T$, where T is the period of the orbit), then:

$$\text{Gravitational Force} = G\frac{Mm}{r^2}, \text{ and}$$

$$\text{Centrifugal Force} = m\omega^2 r = \frac{4\pi^2 mr}{T^2}.$$

Setting the two equal and canceling out the common term m gives:

$$\text{Kepler's Law: } T^2 = \frac{4\pi^2 r^3}{GM}.$$

Notice that the orbital period does not depend on the mass of the planet (the object in orbit), but only on the mass of the Sun (the object it orbits about). The equation gives Kepler's Third Law directly.

As an example, we can use the mathematical expression of Kepler's Third Law to determine the mass of the Sun, given the distance of the Earth from the Sun and the period of the Earth's orbit. For example, taking $r_{e\text{-}s}$ = 150 million kilometers, $G = 6.67 \times 10^{-11}\ m^3 / g\text{-}s^2$ and T_e= 1 year (3.15×10^7 sec), the mass of the Sun must be:

$$M_{\mathrm{Sun}} = \frac{4\pi^2 r_{e-s}^3}{GT^2} = \frac{4\pi^2(150\mathrm{x}10^6\mathrm{x}10^3)^3}{(6.67\mathrm{x}10^{-11})(3.15\mathrm{x}10^7)^2} = 2.01\mathrm{x}10^{30}\ \mathrm{kg}.$$

Note this actually depends on a hard thing to measure on Earth — the distance between the Earth and Sun. Kepler's Law relates ratios of radii to ratios of periods or revolution, and 19th century astronomy used the transit of Venus across the sun to determine the Earth-Sun distance, from which all other distances could be determined.

Problem Set 2a: Kepler's Laws

Problem 1: Calculate the mass of the Earth from the period of revolution of the moon around the Earth. The average orbital distance between the Earth and moon is 3.8×10^5 km. If the radius of the Earth is 6.37×10^3 km, what is the average density of the Earth?

Problem 2: Calculate the density of Jupiter given that Jupiter's equatorial radius is 71,492 km and that Ganymede, one of Jupiter's moons, orbits Jupiter at an average distance of 1.07 million km every 7.15 days.

Problem 3: Why is Jupiter so much less dense than the Earth?

2.2 Energy Fluxes and Events on the Early Earth

This section reviews the concepts of force, work, energy, units of measurements and also illustrates the use of dimensional analysis. Our purpose is to lay a foundation for energy calculations such as:

(1) Heating all the dust and agglomerated rock fragments that collected to form the Earth - to the melting point.
(2) Relative energy flux in solar radiation and meteorite bombardment.

Force

Force can be defined by Newton's Law:

$$\boldsymbol{F} = m\boldsymbol{a},$$

Where $\boldsymbol{F}$ is the force in Newtons (in the MKS or meter-kilogram-second system, also known as SI) or Dynes (CGS or centimeter-gram-second system), m is the mass in kilograms (MKS) or grams (CGS), and $\boldsymbol{a}$ is the acceleration in m/sec^2 or cm/sec^2. A Newton clearly has the the dimensions of kg-m/sec^2.

Why? Show that a Newton is 10^5 dynes.

Work (Energy)

By definition, work or energy, E, is the product of force and distance. If the force is not constant, E may be defined:

$$E = \int_{r_1}^{r_2} F \cdot dr$$

Where E is energy in Newton-m, also called Joules, $\boldsymbol{F}$ is force in Newtons, and dr is incremental (radial) distance in m. If r_1 is taken to be ∞ and r_2 the surface of a planet, E is the gravitational potential energy of a distant stationary body with respect to the surface of that planet. If that body falls to the surface of the planet, this gravitational potential energy is converted to kinetic energy. When the body hits the surface the kinetic energy is transferred to smaller objects (explosion) and ultimately converted to heat, some or much of which is radiated back into space. By definition energy is given the units of Joules (1 Joule = 1 N-m in SI or MKS)

Show that energy has the units kg-m^2/sec^2.

Energy Equivalents

To develop an intuitive understanding it helps to express energy in different units. The relationships between some convenient units are:

1 joule = 1 kg-m^2/sec^2 = 10^7 ergs = 10^7 g-cm^2/sec^2

1 calorie = 4.186 joules Historical note – a calorie was defined as the amount of heat (then tought of as 'caloric') that would raise the temperature of 1g of water by 1°C, before it was realized that heat and work are both forms of energy. Since chemical manipulations still frequently use calories, while physical ones are in SI joules, this is one conversion that sometimes can't be avoided. However, it is convenient that roughly a joule of energy is needed to heat a gram of silicate mantle by one degree Kelvin. (When calories were being defined, it was also not yet known how unusually high water's heat capacity was relative to most other solids and liquids...)

1 ton TNT = 4 x 10^9 joules = 4 GJ

Energy produced by a large 100MW powerplant in one year= 3.15x10^{15}J

Largest historical earthquake ~ 10^{19}J (Chile, 1960 – Magnitude 9.5, with ~800km of surface rupture)

Solar flux at distance of Earth from Sun = 1.34 kW/m^2 (1340 J/m^2-s)

Planetary data you will find useful include:

Planet	**Radius (10^6 m)**	**Mass (10^{24} kg)**	**Distance from Sun (10^{11} m)**
Mercury	2.44	0.33	0.58
Venus	6.05	4.9	1.08
Earth	6.38	6.0	1.50
Mars	3.4	0.64	2.29

Newton's Law of Gravitation

Newton's law of gravitation is:

$$\vec{F}_{gravity} = \hat{r}\, G\frac{(M_1 m_2)}{r^2},$$

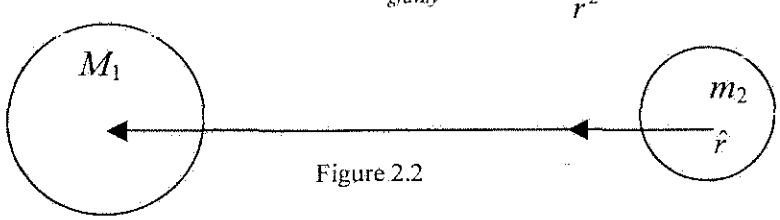


Figure 2.2

Where G = 6.67 x 10^{-11} N-m^2/kg^2 is the gravitational constant.. M_1 and m_2 are the masses of the two objects attracting one another, r is the distance between the centers of masses of the two objects, and $\hat{r}$ is a unit vector pointing from the test mass m_2 toward M_1 (Figure 2.2).

Show that G has units of $m^3/kg\text{-}sec^2$.

For convenience, gravitational acceleration, g, is defined:

$$\vec{g} = \hat{r}\,\frac{GM_1}{r^2},$$

so that:

$$\vec{F}_{gravitational} = m_2\vec{g}$$

This expression is a close analog to Newton's definition of force defined above. For the Earth, g is close to 9.8 m/sec^2 from the surface to 4000 km depth.

Calculate $\vec{g}$ for the surfaces of the Earth, Mars and Venus.

How can $\vec{g}$ be constant to half the radius of the Earth? (Answer the question here in words and mathematically in problem 3 below.)

Problem Set 2b: Energy Fluxes

Problem 1: Answer all the questions in italics in Section 2.2 above.

Problem 2: Calculate the solar flux to the Earth (e.g., how much solar energy impinges on a disk of radius equal to the Earth's at the distance of the Earth from the Sun?)

Problem 3: If the gravitational attraction of mass at radii (from the center of mass) greater than a point buried within a planet cancel (they do), the gravitational attraction at a point in the interior of a planet is due only to that material in the spherical shells of material below that point. In other words, if I measure gravity at a point 4000 km below the Earth's surface (2380 km from the center of the Earth), the gravitational attraction is as if I were on a planet 2380 km in radius. Use this insight to calculate the density of the Earth below 4000 km depth, if the gravitational acceleration, $\vec{g}$, equals 9.8 m/s^2 at that depth.

Problem 4: The mass of the Earth's iron-nickel core is about 1/3 that of the entire Earth. If this metallic material was initially mixed uniformly in the newly accreted Earth and then segregated to the center of the Earth, how much gravitational energy would be released? Make a rough estimate assuming (on average) the iron-nickel mass moved through a constant gravitational field of 9.8 m/s^2 from 2/3 r_e to 1/3 r_e. What would this energy release mean for the heat budget of the early Earth? If all of the energy went to heat, how much would the Earth be heated? Assume the heating was uniform and that the heat capacity of the Earth is 1 J/g-K (i.e., assume that it takes 1 Joule to warm 1 gram of Earth material by 1°C). How many millions of years of solar flux would be equivalent to the energy released in this "iron catastrophe"?

Problem 5: Walter and Louis Alvarez have suggested that periodic collisions of the Earth and 20 km diameter meteorites are responsible for major extinctions such as that of the dinosaurs at the Cretaceous-Tertiary boundary 65 million years ago. A 20 km diameter meteorite would create a 200 km diameter crater. Estimate the amount of energy released in such an impact by calculating the work gravity does on the meteorite as it falls from a very large distance to the Earth's surface. Assume a meteorite density of 3000 kg/m^3. How many days of solar flux on Earth would equal this energy? Note that this gravitational energy would have been transformed into kinetic energy $\left(K.E. = m_{meteor} v_{meteor}^2/2\right)$ of the moving meteor just before it hit. How fast would the meteorite be moving when it hit Earth's surface? Can you think of any ways it could be moving faster (or slower?) than this?

Problem 6: How many minutes of Sunlight on the area 50,000 m^2 (the area of a large powerplant complex) would equal the energy produced by a 100MW powerplant in a year? Compare the energy (in terms of millions of years, days, or minutes of Sunlight) of the segregation of Earth's core and the impact on Earth of a 20 km meteorite.

Problem 7. Making planets by colliding ice-cubes. The Earth is presumed to have grown by incessant collisions of smaller objects. Imagine a small world of Earth-like density, upon which small comets (ice-cubes) rain down, pulled in by the proto-Earth's gravitational attraction. At what earth-radius would the kinetic energy release from such a collision generate more heat that the latent heat it would take to melt the impacting ice-cube? At what radius would the impact release enough energy vaporize the impacting ice-cube? (Assume each comet is composed of ice at 273K, only considering the enthalpy of melting of ice, which is 333kJ/kg. The enthalpy of vaporization of water at 373K is 2222kJ/kg. Also assume that all of the impact energy goes into heating the impacting ice-cube, not the proto-Earth)

Problem 8: Energetics of a moon-forming impact. The current conventional wisdom for the formation of the moon is that roughly ~30Ma after accretion of most of the Earth (and segregation of most of the core), a large Mars-sized planet collided with the proto-Earth, the impact leading to the formation of the moon from the ring of ejecta induced by this impact that collected at a distance greater than 3 Earth radii. Assume a Mars-mass object collides with an Earth-mass object at the speed predicted by their maximum gravitational potential energy. To what temperature would this energy heat the Earth, if after the impact, the heat were uniformly distributed throughout the proto-Earth, assuming the heat capacity of the Earth is ~1 kJ/kg.) If all of the gravitational energy release went into the impacting object, do you think it would melt or be vaporized? (The enthalpy of melting of an Earth-like silicate is ~660kJ/kg, while the enthalpy of vaporization is ~13 MJ/kg.)

Problem 9: Discuss in a paragraph or two (not more) the role that "back of the envelope" calculations such as you have carried out above play in comprehending the past and future history of the Earth? How do such calculations compare to more sophisticated and accurate computer simulations? Why are "back of the envelope" calculations still useful in the computer age?

2.3 The Black Body Temperature of Planets

After the marked density contrast between the inner and outer planets, the most surprising observation on our solar system is probably the difference in temperature between the Earth and its nearest neighbors. Venus, the next closest planet to the Sun has surface temperatures high enough to melt lead. Mars, the next farther away, has surface temperatures sufficient to make dry ice (CO_2) freeze. Are the differences simply due to the distance from the Sun? Calculating the black body temperatures of the planets allows this possibility to be evaluated and is the first step toward understanding planetary temperature.

A perfect black body radiates according to a simple law discovered by Plank:

$$R_C = \sigma T^4$$

Where R_C is the radiance in J/m^2-s (or, equivalently, W/m^2), σ is the Stephan-Boltzmann constant with a value of 5.65 x 10^{-8} J/m^2-sec-K^4, and T is temperature in degrees Kelvin (= °C + 273.15).

Example 1: Given that the flux of solar heat to the Earth at the mean distance of the Earth from the Sun is 1.34 kJ/m^2-s, and that the radius of the Earth is 6.37 x 10^6 m, use Plank's Law to calculate the black body temperature of the Earth.

Note that solar energy is intercepted by the disk the Earth presents to the Sun (area = πr_e^2) whereas, because the Earth rotates, the Earth radiates energy from its full spherical surface ($4\pi r_e^2$). Thus if the Earth's albedo (the fraction of incident energy reflected to space) is A we can write the following general equation for the black body temperature of the Earth or any other planet:

$$T_{Black\ Body}\left[{}^\circ\mathrm{K}\right] = \left[\frac{F(1-A)\pi r^2}{4\pi r^2 \sigma}\right]^{\frac{1}{4}} = \left[\frac{F(1-A)}{4\sigma}\right]^{\frac{1}{4}},$$

Where F is the solar radiation flux in cal/cm^2-sec, A is the albedo of fraction of incident radiation reflected, and s is Boltzmann's constant. If A=0 (no energy reflected),

$$T_{black\ body}[{}^\circ\mathrm{K}] = \left(\frac{\left(0.032 cal/cm^2-\sec\right)\pi r_e^2(1-A)}{\left(1.35x10^{-12} cal/cm^2-\sec-{}^\circ K^4\right)4\pi r_e^2}\right)^{\frac{1}{4}} = 277.45\ {}^\circ\mathrm{K}.$$

$$T_{black\ body}[{}^\circ\mathrm{C}] = 277.45\ {}^\circ\mathrm{K} - 273.15 = 4.3\ {}^\circ\mathrm{C}.$$

Example 2: Determine the Sun's surface temperature from the size of the sun's disk in the sky and the assumption that its blackbody radiation controls Earth's surface temperature. The angular diameter of the Sun in the sky is about 0.5° or 8.72×10^{-3} radians, so its angular area is that of a disk $\left(area = \pi * radius^2\right)$ of radius 4.36×10^{-3} radians, i.e. an angular area of 6.05×10^{-5} steradians. The total angular area surrounding the Earth is 4π steradians, so the blackbody energy balance determining the surface temperature of the Earth is that the incoming solar radiation energy from the Sun's disk $\left(A_{sun-disk}\right)\sigma T_{sun-disk}^4$ is equal to the outgoing terrestrial radiation energy radiated in all directions $4\pi\sigma T_{earth-surface}^4$. Balancing these gives $6.05x10^{-5}\sigma T_{sun-disk}^4 = 4\pi\sigma T_{earth-surface}^4$ or $T_{sun-disk} = \left(4\pi/6.05x10^{-5}\right)^{1/4} T_{earth-surface} = 21.4 T_{earth-surface}$. For a blackbody Earth surface temperature of 277K, this would predict the Sun's surface temperature to be 5930K, in good agreement with the ~6000K solar surface temperature that is consistent with its yellow color. So we can indirectly determine the Sun's mass from the Earth's orbital period, and the Sun's surface temperature by Earth's surface temperature.

Problem Set 2c: Black Body Temperature

Problem 1: Given that the flux of heat from the Sun is inversely proportional to the square of the distance from the Sun, use the following table to calculate the black body temperatures of Mercury, Venus and Mars.

Planet	Distance from Sun	Solar Flux	Black Body T
Mercury	58×10^6 km	?	?
Venus	108×10^6 km	?	?
Earth	150×10^6 km	1340 J/m^2-sec	~4.3°C (~277.6K)
Mars	228×10^6 km	?	?

Problem 2: What is the physical meaning of the black body temperature of a planet?

Problem 3: What would the average black body temperature of the side of the Earth always facing the Sun be if the Earth rotated like the moon, once per orbit? Is your answer reasonable?

Problem 4: The Earth also loses internal energy produced by radioactive decay, with an present-day average surface heatloss of roughly 70 mW/m^2. (Earth's heatflow varies from ~25mW/m2 to >300mW/m2, being lowest in the oldest parts of continents, and highest where two plates are spreading apart.) What fraction of the solar flux is Earth's internal surface heatloss? Continuing the exploration in the above problem, what would the temperature of the 'Dark side of the earth' be if it lost this internal heat while always facing away from the sun?

Problem 5: Over the last 4.5 billion years, the solar heat flux has probably increased by about 20%. Given that the Earth's black body temperature is now 4.3°C, what should the Earth's average temperature have been, if it has increased as the black body law suggests it should?

Problem 6: Temperatures in the Earth's tropics. Sunlight falls more nearly vertically in Earth's tropics, thus heating them more than the polar regions. To estimate the appropriate blackbody temperature for Earth's tropics, assume the equatorial band is a rotating disk in local equilibrium. The tropical belt has area $2*\pi*r_E*W_{EQ}$, where W_{EQ} is the width of the equatorial tropics. Since the tropical belt receives solar energy over an area $2*r_E*W_{EQ}$ and radiates energy over an area $2*\pi*r_E*W_{EQ}$, what is its predicted blackbody temperature? Does this answer seem reasonable? Why or why not?

Chapter 3: The Age of the Earth and the Geologic Record

Perhaps the most fundamental and venerable question in the Earth Sciences is a very simple one: How old is the Earth and Solar System? Every educated person should know the answer to this question and understand the logic upon which it is based. Radiometric dating techniques provide a remarkably accurate and consistent answer: 4.56 billion years.

The simplest method of dating the Earth is to date the carbonaceous chondrites that accreted and then collected to form the Sun and Earth and other planets of our solar system. Radiometric dating measures the number of daughter atoms created by the decay of a radioactive parent. The more parent atoms, the more daughters produced in a given period of time. Minerals in any natural sample will have different amounts of parent (radioactive) material, and hence will produce different numbers of daughters. For simplicity, we measure the ratio of the present number of parent atoms and the present number of daughter atoms to the number of atoms of a stable isotope of the daughter. For example, if the parent is ^{87}Rb, the daughter ^{87}Sr, and the stable isotope ^{86}Sr, we measure ^{87}Rb /^{86}Sr, and ^{87}Sr /^{86}Sr. A key assumption in the radiometric method is the assumption that the initial ratio of ^{87}Sr /^{86}Sr has been homogenized by the event that set the radiometric clock. This assumption can be verified by the coherency of the data and appears to almost always be a good assumption. Given at least two measurements of ^{87}Rb /^{86}Sr and ^{87}Sr /^{86}Sr, the dependence of daughter production on parent concentration can be determined and the age of the sample measured.

The essential concept can be appreciated by a simple thought experiment. Imagine that during the Second World War all the children of England were sent to farms for safety. The children were distributed such that each farm had exactly 2 children at the time the redistribution was ordered. Suppose different farms had different numbers of parents and that every pair of parents produced one additional child every 2 years. If all children stayed on the farm, you could clearly tell the time since the redistribution by plotting the number of children per farm against the number of parents per farm. At t=0 all farms would have 2 children. At t=2 years farms with 2 parents would have one additional child and a total of 3 children, those with 4 parents 4 children, etc. In 4 years farms with 2 parents would have 4 children, farms with 4 parents 6 children, etc. The number of children per farm plotted against the number of parents per farm would always be a straight line. At t=0 the line would be horizontal, at t=4 years it would have a slope of 1, etc. The children per farm will increase linearly with the number of parents per farm. If you know the number of parents and the number of children on a farm and the number of children initially there, you know the time since the redistribution.

This is the secret of radiometric dating. The parents correspond to the radioactive atom, say ^{87}Rb. The children correspond to the daughter products of radioactive decay. ^{87}Rb decays to ^{87}Sr. ^{87}Sr is the daughter product. The farm corresponds to the stable isotope of the daughter element, in this case ^{86}Sr. When a rock melts (the redistribution or homogenizing event) the ratio of ^{87}Sr /^{86}Sr is homogenized in all minerals and all samples of the melted material, just as equal numbers of children were sent to each farm in our hypothetical example. *This ratio is the same anywhere you could measure it.* Homogenization occurs because high temperature chemical reactions do not discriminate between different isotopes of the same element, they just discriminate between different elements. (This occurs because different isotopes have the same atomic size and the same electron bonding structure, while only differing in mass by a percent or so.). Typically, different minerals will contain different amounts of Rb, and different samples of the melt will contain different mixes of minerals and hence different amounts of ^{87}Rb and different ratios of $^{87}Rb/^{86}Sr$. There are different numbers of parents per farm, but initially there are the same number of children per farm. Measuring the daughters per farm and the parents per farm on a number of farms with different numbers of parents will tell the time that has elapsed since the homogenization event.

The mathematics that show this for radioactive parents (parents that can transform into daughters) is presented in the following section.

3.1 Radiometric Dating

Let P be the number of radioactive (Parent) atoms in a sample. The probability of decay is proportional to P (we choose a proportionality constant λ) and time, t:

(1)

$$dP = -\lambda P\, dt$$

$$\frac{dP}{P} = -\lambda\, dt$$

$$\int_{P_0}^{P} \frac{dP}{P} = \int_0^t -\lambda\, dt$$

(2a) $$\ln \frac{P}{P_0} = -\lambda t$$

Where P_0 is the initial number of parents at $t = 0$.

If $t_{1/2}$ is the time at which $P/P_0 = 1/2$, then:

(2b) $$t_{1/2} = \frac{-\ln(1/2)}{\lambda} = \frac{\ln(2)}{\lambda} = \frac{0.6931}{\lambda}$$ and

(3) $$\frac{P}{P_0} = e^{-\lambda t} = e^{-0.6931 t / t_{1/2}}$$

Let D represent the number of atoms that are the product of radioactive decay of a parent atom (called daughter atoms, e.g., ^{87}Sr). Let D_0 be the number of daughter atoms immediately after the homogenizing event at $t = 0$, and S be the number of stable atoms of the same element. Stable here means that S is not the product of radioactive decay. Both D and S are non-radioactive. Then:

$$D = D_0 + P_0 - P.$$

$$P_0 = \frac{P}{e^{-\lambda t}} = Pe^{\lambda t}$$

$$D = D_0 + P\left(e^{\lambda t} - 1\right), \text{ or}$$

(4a) $$\frac{D}{S} = \frac{D_0}{S} + \frac{P}{S}\left(e^{\lambda t} - 1\right)$$

(4b) $$\frac{D}{S} = \frac{D_0}{S} + \frac{P}{S}\left(e^{0.6931 t/t_{1/2}} - 1\right)$$

(4c) $$\left[\frac{^{87}\mathrm{Sr}}{^{86}\mathrm{Sr}}\right] = \left[\frac{^{87}\mathrm{Sr}}{^{86}\mathrm{Sr}}\right] + \left[\frac{^{87}\mathrm{Rb}}{^{86}\mathrm{Sr}}\right]\left(e^{0.6931 t/t_{1/2}} - 1\right) \text{ or}$$

$$\left[\frac{^{87}\mathrm{Sr}}{^{86}\mathrm{Sr}}\right]_{now} = \left[\frac{^{87}\mathrm{Sr}}{^{86}\mathrm{Sr}}\right]_{orig} + \left[\frac{^{87}\mathrm{Rb}}{^{86}\mathrm{Sr}}\right]_{now}\left(e^{\lambda t} - 1\right)$$

Problem Set 3: Radiometric Dating

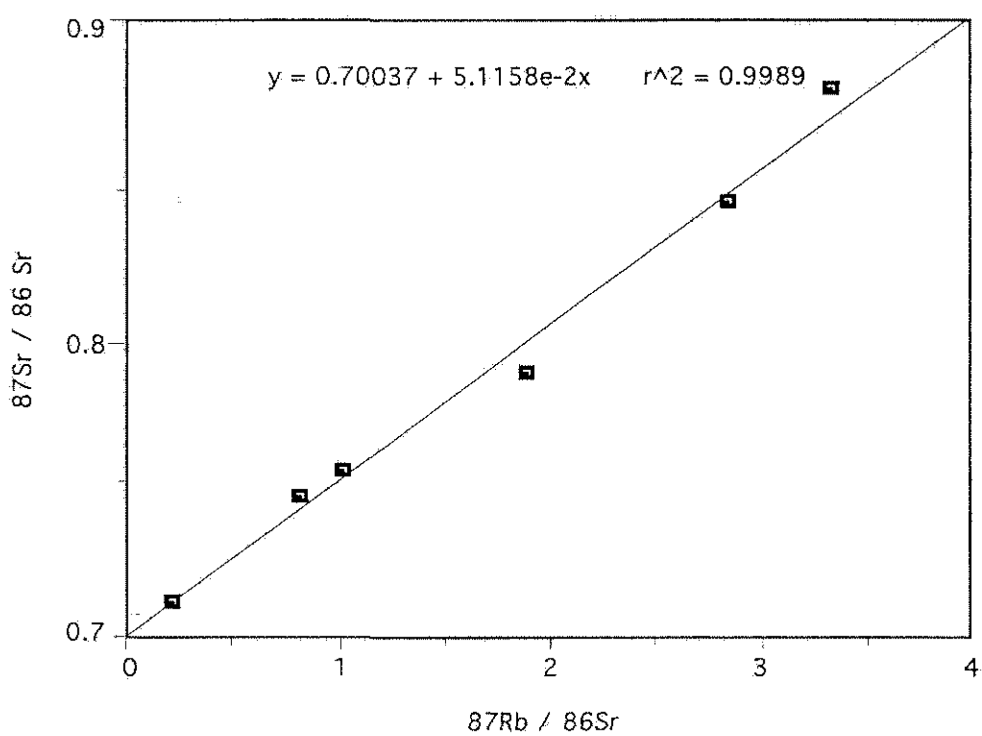
Problem 1: Rewrite (4c) in terms of $(^{87}Rb/^{86}Sr)_{orig}$ instead of $(^{87}Rb/^{86}Sr)_{now}$.

Problem 2: If the half-life of ^{87}Rb is 47 Gy, what is λ for ^{87}Rb?

Problem 3: Whole rock samples collected from the Amitsog gneiss in the Quilangarssuit area of southwestern Greenland have the $^{87}Rb/^{86}Sr$ and $^{87}Sr/^{86}Sr$ values shown below. The data are plotted below. A line has been "fit" or regressed to the data. Its equation is given at the top of the plot. Note "5.1158e-2x" means 5.128 x 10^{-2}, and "r^2" is the r-squared measure of the "goodness of the fit", with 1.0 being a perfect fit. What was the initial $^{87}Sr/^{86}Sr$ value? Is this close to the primordial (meteorite) value of .700? Is this reasonable? Why? These gneisses consist of meta-volcanic and meta-sedimentary rocks including banded ironstones and conglomerate. What does this mean about conditions on the Earth at the time these samples were metamorphosed and the radioactive clock reset?

$^{87}Rb/^{86}Sr$	$^{87}Sr/^{86}Sr$
0.2	0.712
0.8	0.749
1.0	0.754
1.9	0.787
2.6	0.830
3.25	0.874

Amitsoq Gneiss, Quilangarssuit Area Greenland



Problem 4: A suite of whole rock samples was collected in the Archean shield (or basement rock older than 2.5 Ga) near the very large Sudbury deposit. Determine the age of the intrusives from the plot.

Whole rock isochron from granites and Gabbros collected near Sudbury, Ontario

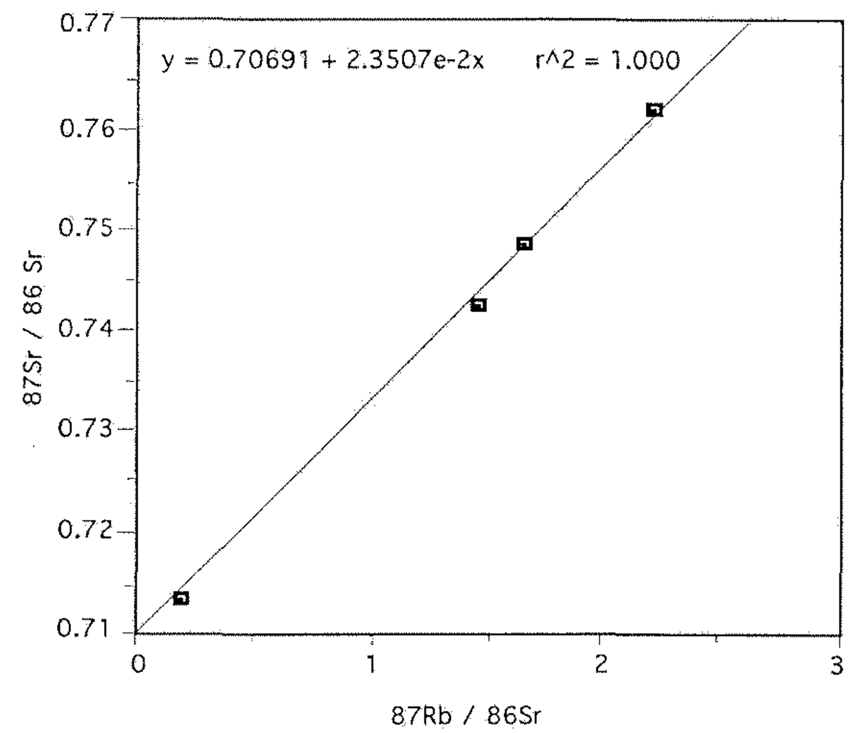


Diagram for Problem 4

Problem 5: The initial ratio of $^{87}Sr/^{86}Sr$ differs in the very old Greenland samples and the Sudbury intrusives (problem 4). The Greenland samples have initial values of $^{87}Sr/^{86}Sr$ very close to the chondrite value of 0.700. The Sudbury samples have a much higher initial ratio. What can you infer from this observation about the separation of sialic (Rb-loving) material from the mantle?

Problem 6: Samples from a stock (formerly hot igneous intrusive rock mass) in the Archean shield have been collected for Rubidium-Strontium dating. Different minerals were analyzed as indicated below. What is the age of the rock? How long ago did the Archean Eon end? What can you say from your calculation about the intrusive event?

Mineral A:	$^{87}Sr/^{86}Sr = .740$	$^{87}Rb/^{86}Sr = 1.18$
Mineral B:	$^{87}Sr/^{86}Sr = .758$	$^{87}Rb/^{86}Sr = 1.73$

Chapter 4: Geology - The Story of the Earth's Last 4.56 Ga.

For the first 1.5 Gy (billion years) or so since 4.56 Ga, the Earth was intensely bombarded by meteorites and comets. During this time it lost essentially all of its atmosphere at least once, during the moon-forming impact. (In contrast, moonless Venus retained much of her original heavy volatile atmophile elements such as the rare gas Argon). The record preserved on the moon's surface shows that the period of heavy bombardment ended at roughly ~3.9Ga

The Earth quickly achieved a state similar to the one it now has. The oldest known rocks on Earth are 3.96 billion years old and are sediments with volcanic fragments found in Canada. The existence of sediments this old means that since very early times there was flowing liquid water on the Earth's surface. (The oldest zircon mineral so far found in sediments is a 4.2Ga zircon from Australia. In striking contrast, of the 7 near-random points on the moon from which rock samples have been collected and brought back to Earth to be dated, rocks as old as 4.3Ga have been found.) Furthermore, over most of the last 83% of the Earth's 4.56 billion year history the temperature of the Earth has been within a narrow band where the oceans have neither frozen nor boiled. From isotopic and fossil evidence, the temperature of the Earth has been within a few tens of degrees of its present values for most of its ~4 billion years. If anything, the Earth has cooled slightly with time despite a 10 to 20% increase in the radiancy (watts of radiant energy emitted per m^2 of the solar surface) of the Sun. The exception to this generalization may have been the late Proterozoic when the oceans did freeze completely about 4 times and the Earth oscillated between super-icehouse (-50°C) and super greenhouse (+50°C) conditions. This is the "snowball Earth" hypothesis of Kirschvink, recently championed by Hoffman and Schrag. These drastic temperature variations posed a dramatic challenge to early life. They may have forced the development of super-adaptable species, and prepared the way for the explosion of different life forms in the Phanerozoic eon that followed.

Sedimentation has operated on Earth for the last 4 Ga. So what? The fact is sediments provide a key window into the Earth's past. Herodotus, a Greek philosopher, was one of the first to realize the potential of sediments for reaching into the distant past. In 450 BC he discussed the shape of the Nile delta, and commented that such a large pile of sediments must have taken a very long time to accumulate from the few centimeters deposited each year during the annual flooding of the Nile. Herodotus' characterization of the form of the river sedimentation as a Greek Δ has continued to this day in the word river "delta". Sedimentary basins, the areas where sediments accumulate, are the Earth's garbage cans, areas where the products of erosion, dead animals, shells and bones are deposited in a time-ordered stack with the oldest units on the bottom and the youngest on top. Geologic history, like human archeology, is told largely in terms of who has died and what nature has thrown away. The study of fossil remains in sedimentary sequences, now dated by the radiometric methods discussed in Chapter 3, provides a fascinating view of how life evolved on the Earth. Changes in evolution provided the initial basis for dividing geologic time into periods. Even before it was realized that fossils are remains of dead animals, the appearances and disappearances of different fossil types (e.g. trilobites, dinosaurs) was being used to mark relative ages.

In working out a geologic time scale, geologists did just what anyone would do. They put boundaries where there were recognizable changes in fossils (Figure 4.1). All the geologic boundaries are marked by distinctive evolutionary changes. The division of the geologic time scale thus tells us about extinctions, and the evolutionary discontinuities caused by the extinctions. The most important and longest divisions are called "eons" and "eras". The next most important are "periods", and the finest divisions are "epochs".

Although early in the Earth's history there were no hard-shelled fossils, microfossils of early life forms can be detected under a microscope. From such evidence we know that life probably started about 4 billion years ago. Bacteria and algae were well established by 3.8 Ga (billion years ago). By about 2.5 Ga they were well enough established that they produced, through photosynthesis, an oxygen-rich atmosphere similar to the one we have today. The oxygen was released as a by-product or waste that those organisms disposed of. The interaction of that "poisonous" (or oxygenating) atmosphere with the barren rocks of the continents leached iron in massive quantities which accumulated in almost pure form in shallow seas, forming (about 2.5 Ga) what still remain the Earth's richest, most extensive, and most valuable iron deposits. The Mesabi Range in Michigan is one of these Superior-type sedimentary iron deposits. Early or Precambrian time is divided into the Archean Eon (or pre-oxygenating atmosphere, to 2.5 Ga) and Proterozoic Eon (post oxygenating atmosphere). The eon of shelled fauna (from the time they developed to present) is called the Phanerozoic Eon. The Phanerozoic Eon is divided into the Paleozoic, Mesozoic, and Cenozoic Eras.

The first shells appear in the first period of the Paleozoic Era. The Cambrian Period ran from the end of the Proterozoic at 544 Ma to the Ordovician Period, which started 505 million years before present. Shortly before the Cambrian, there were soft bodied worms and jellyfish-like creatures called Ediacaran fauna. Marine invertebrates (graptolites, trilobites) dominate the fossil record of the Ordovician, which lasted from the end of the Cambrian until 440 million years ago. Fish appeared in the Silurian Period that lasted from 440 to 410 Ma. Land plants and trees evolved at the end of the Devonian (why not sooner?), reptiles at the end of the Carboniferous, and the mega-continent Pangea was assembled in the Permian (286-251 Ma). The time from Cambrian to Permian is called the Paleozoic Era. The names of the Paleozoic periods can be remembered with the phrase "Come Over Some Day Maybe Play Poker" for Cambrian, Ordovician, Silurian, Devonian, Mississippian, Pennsylvanian and Permian. In the table below the Carboniferous period includes both the Mississippian and Pennsylvanian periods. The end of the Permian marks the start of the Mesozoic Era. The Mesozoic Periods (Triassic, Jurassic, and Cretaceous) saw the peak development of the dinosaurs and the evolution of mammals. Almost all species preserved in the fossil record are now extinct; only 0.1% of all species that have lived on Earth live on present-day Earth.

Eon	Era	Period	Epoch	10^6 yrs	Appearance
Phanerozoic	Cenozoic	Quaternary	Holocene	0.01	Humans
|			Pleistocene	1.8	Ice ages
|					
|		Tertiary	Pliocene	5.0	Cool, kelp, grasslands
|			Miocene	23	Grasses
|			Oligocene	38	Mammals
|			Eocene	57	Global cooling
|			Paleocene	66	Extinction of Dinosaurs
|	Mesozoic	Cretaceous		146	Dispersion of Pangea
|		Jurassic		201	First Birds
|		Triassic		251	Reptiles
|					
|	Paleozoic	Permian		286	Pangea
|		Carboniferous		360	Coal, large insects
|		Devonian		410	Land Plants
|		Silurian		440	Fish
|		Ordovician		505	Gondwana
		Cambrian		544	Shelled Organisms
				570	"jellyfish" Multicellular life (Ediacara)
Proterozoic				2500	Oxygen Atmosphere
Archean				3800	1st Primitive Life
				4560	Age of Earth

Figure 4.1 Geologic Time Scale (http://www.ucmp.berkeley.edu/help/timeform.html)

A major change occurred at the end of the Mesozoic Era, at the Mesozoic-Cenozoic boundary, or as it is often called, the Cretaceous-Tertiary boundary. The Tertiary is the first Cenozoic Period. At this time, ~66 million years ago, the dinosaurs "suddenly" became extinct, and small, hairy, warm-blooded mammals that had been evolving in coexistence with dinosaurs for over ~100Ma inherited the Earth, replacing dinosaurs as the largest grazing animals and predators. Initially there was much debate about the cause or causes of this important extinction event (and evolutionary opportunity). Some believed the extinctions at the Cretaceous-Tertiary (or K-T) boundary were caused by a large meteorite hitting the Earth. Others believed the cause was particularly intense volcanic eruption (the outpouring of the Deccan Traps or flood basaltic lavas in India) associated with normal plate tectonic processes. The debate still rumbles, but has been settled for most with the discovery of the Chixalub impact site off the Yucatan Pennsula.

Not all extinction events are due to impacts. In fact this cause may be relatively rare. For example the extinctions that occurred at the Permian-Triassic boundary at 251 Ma (million years) were almost all marine invertebrates, and the extinctions were diachronous - they occurred over an extended period of time and did not occur all at once, as the meteorite impact hypothesis would suggest. The Permian-Triassic extinction was the largest extinction that has ever occurred. There is a beautiful exhibit covering this extinction in the display case on the ground floor of Snee Hall.

The Cenozoic Era is divided into the Tertiary and Quaternary Periods. The Tertiary Period is divided into the Paleocene, Eocene, Oligocene, Miocene, and Pliocene Epochs, and the final Quaternary Period is divided into the Pleistocene (or glacial epoch) and the Holocene (or post-glacial epoch), which started 10,000 years ago.

The span of geologic time is difficult to grasp intuitively. By 2.5 Ga (billion years) life had developed sufficiently to produce an oxygenating atmosphere. It took until ~0.57 Ga (the start of the Cambrian era) for creatures with skeletons to develop. From then on changes were rapid. Species tended to evolve from small to all sizes including very large, and then, at major extinction events, back to small. The first hominoids made their appearance about 0.005 Ga ago, and recorded human history began about 4000 BC or 0.000006 Ga ago. If the 4.56 Ga evolution of the Earth is taken as one day running from 12 midnight to 12 midnight, the Cambrian era started at 8:58 p.m. (3 hours ago), the dinosaurs became extinct 21 minutes ago, the first hominids appeared 1 minute before midnight, and recorded history started one tenth of one second ago.

Clearly, a historical record of 0.1 second is not much of a basis for projecting what the past day was like. The litany of all the disasters and favorable conditions that have affected humans is not much of a basis for projecting what the Earth has been like or will become. To address such questions we must extend our horizons by using our intelligence and deductive powers to infer what has happened on Earth before we were able to keep written records. We must study geology, the Earth's own written record. Sherlock Holmes was a great detective because he combined acute powers of observation with superb intellect. A good Earth scientist, like a good detective, must combine both these skills. Either one alone is insufficient. For today's Earth scientist, intellect must include quantitative reasoning. Simple logic is no longer adequate.

Additional Reading

W.S. Broecker, 1985, How to Build a Habitable Planet, Eldigio Press, LDEO Box 2, Palisades, NY, p. 291

II. THE ROCK CYCLE

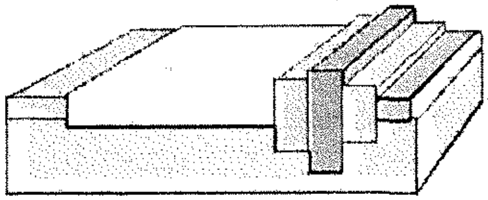
Chapter 5: Earth Kinematics

One of the great secrets the Earth hides from most humans is that it is in actuality a rotating fluid. A clue to this fact is that the Earth is so beautifully round. If it were a solid, the many meteorite impacts it has suffered would certainly have distorted its shape. In fact the Earth is exactly the shape a rotating fluid should have. The Earth's radius at the equator is 22 km greater than at the poles. It is 1/294 out of round, exactly the amount of distortion expected for the Earth's daily rotation rate if the Earth were a fluid. The Earth is fluid (over long timescales) because it is hot (over 1500K at 150 to 200 km depth, and even hotter in its deeper interior. To be sure, the very outermost parts of the Earth are solid. The outer surface of the Earth is cold. The outermost part of the Earth is a thermal boundary layer. Caught up in this thermal boundary layer is a silica-rich sludge we call continents. The thermal boundary layer is called the lithosphere. It is thin and not very strong, but it is solid (not fluid) and it floats on hotter parts of the Earth that underlie it, and moves around with time like the skin on a pot of boiling milk.

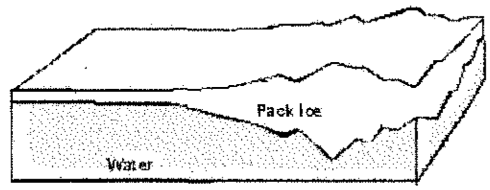
The idea that the 'fit' of Africa and South America means they share a common origin arose almost as soon as the first maps of the globe were produced. Alexander von Humboldt was one of the first to write about the curious pattern in 1801, but suggested its cause was a catastrophic flood — "what we call the Atlantic Ocean is nothing else than a valley scooped out by the sea". The Italian Snider-Pellegrini in 1858 was the first to suggest that the remarkable fit between the West Coast of Africa and the East Coast of South America meant they had split apart, but assumed it occurred in a cataclysmic event. For a long time the matter rested with this kind of speculation. Geologists were aware that the Earth's surface could deform and break, but could not imagine that large horizontal motions could occur. Around 1910 two scientists, Frank Taylor and Alfred North Wegener independently suggested that Africa and South America had split apart. Of these two, Taylor's contribution is largely forgotten – in part because he thought the split happened as a byproduct of capturing the moon 30Ma, an unpopular idea that was criticized even then. His work is, in hindsight, extremely fascinating, as he even noticed the connection between the shape of the coastline of Africa and the shape of what we now call the Mid-Atlantic Ridge, the active plate boundary on which the spreading apart of the two continents has been accommodated (This feature, now seen as the key to understanding the opening history of the Atlantic, was first noted by the Challanger Expedition and Transatlantic cable-laying cruises of the 1870s. Its huge extent was much better mapped by the German Meteor expeditions of the 1920s-1930s, but its importance as a plate boundary would not be recognized until 1960.) Wegener's contribution is now rightly more remembered. Why? Mostly because he did not just present the idea, but made the large effort to synthesize the geologic, fossil, and paleoclimate evidence that the continents had drifted about, joined and split apart, and documented that a great deal of geology and plant and animal evolution can be explained in a very natural and uncomplicated way if this were the case. His ideas and the data supporting them were set forth in a book titled: *The Origin of Continents and Oceans*, first published in 1915 in German, with an English translation of the fourth (1929) edition appearing in 1929. This tour de force combined several then-radical ideas — isostasy (section 6.2) to infer dramatic differences between the continents and seafloor, and drift to explain the geographic, paleoclimate and fossil evolution of the continents. Note too, that like the theory of evolution, this idea was put forward by two people at nearly the same time. Wegener, like Darwin vis a vis Wallace, gets much more credit because he assembled a classic book of arguments to support his case.

If you haven't already, you will learn that new ideas and truth do not always please. Geologists were especially displeased to have their subject explained to them by a meteorologist, and it was not long before this annoyance was voiced. As is also often the case, Wegner's ideas were not criticized broadly in a balanced way, but ridiculed for a single flaw. Sir Harold Jefferies, a formidable mathematical physicist and the leading geophysical theoretician of the time calculated that no conceivable force could propel continental landmasses through solid rock. Wegner's idea of continental drift was simply impossible as anyone with a quantitative education could plainly show. For nearly 50 years Wegener's correct geological and geophysical arguments were academic anathema. Anyone espousing them was viewed as a crackpot by the academic establishment. The concept of continental drift survived in a few "backwaters" such as South Africa and Australia, where geologists were more impressed with the similarity of their geology to that on other continents than they were by mathematical theories that proved continental drift was impossible. A few leading geologists did favor drift, and one, Arthur Holmes, noted in the last chapter of his influential textbook "Principles of Physical Geology (1945)" that mantle convection provided a natural mechanism for continental drift. But "common sense" prevailed, and the hypothesis was alternately ridiculed or neglected, even though Wegener in his book had presented equally powerful arguments against the alternative idea of rising and falling 'land bridges' that had been promoted by paleonologists to explain the fossil evidence. (Jeffries would probably have agreed with Wegener's good physical arguments against land bridges, but thought the point unimportant because he did not see how fossil evidence could say anything important about geodynamics — a common problem in science where specialists discount the value of important observations coming from other disciplines. However Jeffries did clearly state that he had this bias, he was just convinced he was right in his opinions, and argued that since many paleotologists themselves argued about the details of the fossil record in space and time, they must all be confused about the basic fossil patterns that Wegener used as key facts to support his drift scenario. Does this sound like a familiar kind of argument?!)

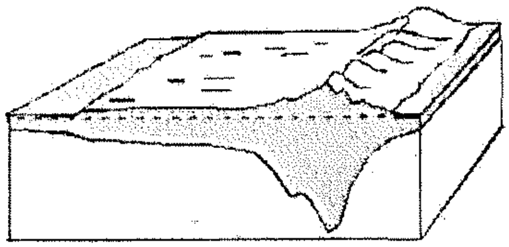
Eventually the evidence became overwhelming. (Now the evidence is completely overwhelming. For example, in the 1970s-80s NASA would have discovered continental drift by accident as a systematic 'drift error' while trying to track orbiting satellites from ground stations. Likewise, in 15 years ranchers in Australia will notice that all their property lines marked with a commercial GPS survey this year are a meter north of the positions measured today.) By 1958, it was generally agreed that the 'Mid-Atlantic Ridge' mountain chain in the middle of the Atlantic Ocean was younger than its flanks. There were no sediments on the tops of the mountain, only very fresh (unaltered) volcanic rock. The rocks in the middle of the oceans were erupted very recently, and decidedly not the oldest material on the planet as had been anticipated. These observations, and also the observation that the deepest parts of the oceans, the oceanic trenches, lay adjacent to volcanic chains, led to the idea of sea floor spreading. The continents need not plough through solid rock, but could drift apart, with upwelling magma forming new ocean floor in the opening gap. An ocean plate could be subducted into the fluid Earth's interior at an oceanic trench to make room for the spreading and new ocean formation between the drifting continents. The answer to Sir Harold Jefferies objection that the continents could not plow through solid rock is that they do not have to: they move like boats caught in an ice flow, with the ice flow, as the ice flow (i.e., the solid lithosphereic shell) is subducted into a trench. The uppermost, most fluid layer of the mantle upon which the lithosphere floats is called the asthenosphere. Isostasy, or floating equilibrium, is illustrated in Figure 5.1.



A) Floating blocks of equal density.
Thicker blocks rise higher and sink deeper



B) Thick pack ice rises higher and sinks deeper than thin ice.



C) High mountains in low-density crust are balanced by a deep root that protrudes into the mantle.

Figure 5.1 Examples of Isostasy.
(Adapted from Earth's Dynamic Systems, p. 44, 1997)

There was direct evidence for this iceberg-like floating equilibrium. It had been known for some time that mountain chains did not produce the gravity anomalies they should if they rested on a rigid substrate, but had just the right anomalies if they were floating like icebergs on a denser fluid below. The Himalayas, for example, do not produce the gravitational deflection of a plumb bob that they might because the positive mass in the towering peaks is offset by the deep light roots of the mountains (see Figure 6.8). The roots are like the 90% of an iceberg that lies beneath the water and supports the impressive ice edifice that rises above the sea surface. The concept of floating equilibrium is called **Isostasy** (Figure 5.1). The outer surface of the Earth or lithosphere floats on the rest of the Earth. Any prism cut through the Earth's surface to a common depth into the mantle contains equal mass. Isostasy is one of the most important general principles in Earth science. It is perhaps the most direct consequence of the fluid nature of the Earth. However, this concept too was only slowly accepted. Even in 1945 an entire book was written by Daly to demonstrate that gravity and topographic surveys of both the US and India (the only such data in 1945) were consistent with regional isostasy.

Sea floor spreading had resolved the mechanical objections to continental drift. An aspect of the problem that had been overlooked proved to be of critical importance. What had been missed was that ocean floor could be created and destroyed by planet-scale convection.

Stratigraphy, the logic of the time ordered sediment stack, again played an important role in leading to a still more surprising discovery. In the 1950's scientists has noticed a curious thing in the ordered sequences of lavas on the main island of Hawaii. Measurements of the remnant magnetism in lava flows showed that the magnetization of certain flows was pointed in a direction opposite to the present direction of the Earth's magnetic field. The scientists had been interested in using these frozen compass directions to study minor variations in the location of the magnetic pole, but the rapid movement of the North Pole to the South Pole was not a minor variation. The flows had not been turned upside down, and it did not appear that rock magnetization could spontaneously reverse direction, although these possibilities were carefully investigated (as these seemed more likely hypotheses than the idea that the Earth's magnetic field has flipped field directions through time!).

The apparent magnetic reversals in the volcano-stratigraphic record were a scientific curiosity to non-paleomagnetitions until 1963 when Vine and Matthews discovered the new oceanic crust was magnetized in normal and reverse polarity stripes running parallel to the mid-ocean ridges (This idea, too, was discovered by Morley shortly before Vine and Matthews, but his submissions to Nature and the JGR were rejected – here it almost certainly helped Vine and Matthews to get published that they were at one of England's most prestigious universities, Cambridge. It also helped that they presented new observational evidence to back up their speculations. After this, Vine went to Princeton for a Postdoc, and while there delivered a series of lectures in many US universities explaining these ideas and the observations that supported them. Again, the strong 'sales' efforts played a big role in convincing others of the truth of these ideas.) The seafloor magnetic stripes would rapidly provide the age-dating tool allowing the history and rate of spreading to be determined, which allowed Morgan and Le Pichon (again, working independently after Le Pichon had heard Morgan's first talk on the concepts of Plate Tectonics) to prove that the oceans were spreading as rigid blocks. The relatively small number (10-12) of rigid plates into which the Earth's outer shell, or lithosphere, is divided - interacts mainly at their edges (Figure 5.2). Volcanoes, Earthquakes and crustal deformation occur at the edges of plates: where the plates slide relative to one another, collide, or drift apart. The "linear" edges of the plates explain why mountains form "chains". The **theory of plate tectonics** resulted from combining the concepts of sea floor spreading (proposed independently by Hess and Dietz), magnetic stripes (Vine-Matthews-Morley), transform faults (Wilson) and near-rigid surface plates with the idea that the Earth's size has remained constant, so that new surface is created at spreading centers and consumed at trenches. (The idea that gravity systematically decreased and thus Earth expanded through time may seem nonsensical now, but was viewed as a serious hypothesis in 1960. Perhaps the simplest argument against it is the lack of expansion cracks on the surface of the moon — it too should have expanded if the Earth had). The ideas are simple, but their proper quantification requires use of the algebra describing motions on a sphere that was developed by Euler. Today it **is the central theoretical framework of geology**. Like the theory of evolution in biology, it is capable of organizing diverse geologic observations and explaining them in terms of Earth processes. One particular merit of the theory for US geologists was that it offered a neat explanation for the volcanic and tectonic evolution of California and the western US.

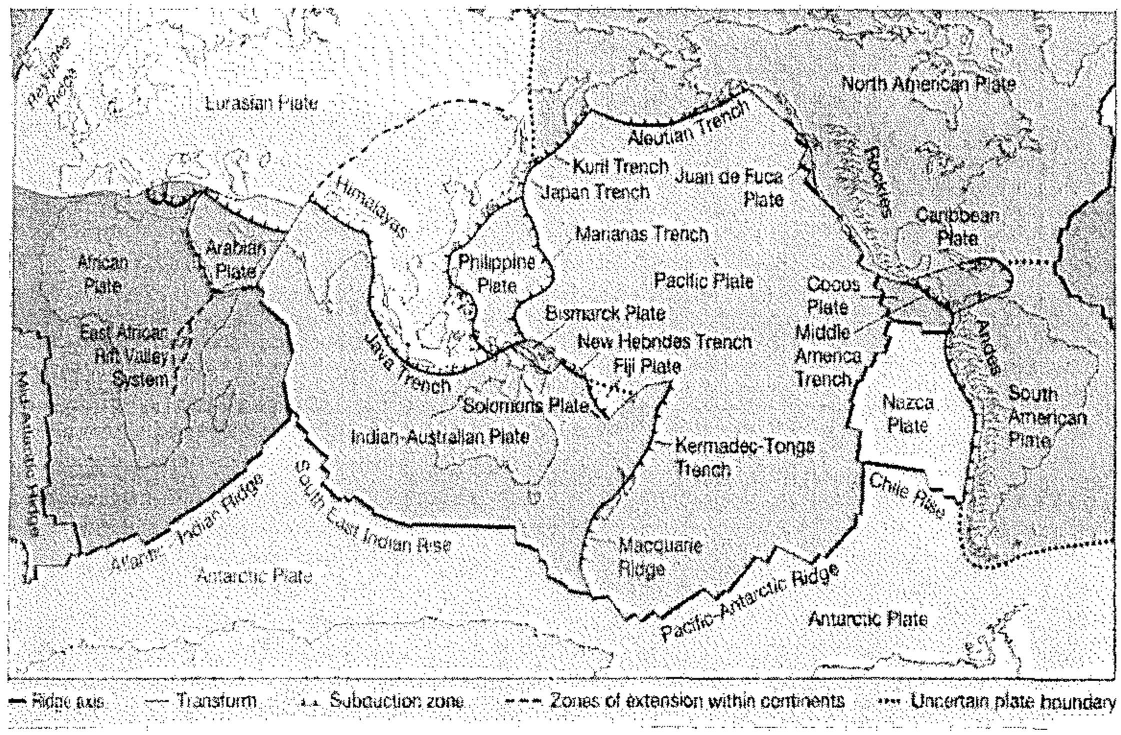


Figure 5.2 - The lithosphere is divided into rigid spherical shell-like pieces or 'plates'. (Adapted from The Earth System, p.111, 1999).

5.1 Plate Tectonics Lab

The concepts of sea floor spreading and plate tectonics were quickly tested and dramatically confirmed by seismologists who showed that Earthquakes literally ring (and therefore outline) the Earth's major crustal plates. Two of these seismologists are at Cornell, Bryan Isacks and Jack Oliver (recently retired).

Two computer programs you will use in this lab nicely illustrate the principles of Plate Tectonics and Sea Floor spreading, and the kind of confirmatory information provided by Earthquakes. To run these programs, you may work in groups of several students. Each group should have at least one member who is familiar with Macintosh computers.

I. Select the "Sea-floor Spreading" program from the Planet Earth folder

Read the introduction. When you have finished, exit the introduction by clicking on the OK box. Then pull down the help menu and select "Concepts of sea-floor spreading". When you have finished reading this pull down the suggestions menu.

1. Select suggestion 3.
   Answer question a.

   Answer question b.

2. Select suggestion 5.
   Answer question a.

3. Select suggestion 6.
   Answer question a.

Pull down the file menu when you are done and select quit.

II. Select the "Epicenter" program from the Planet Earth Folder.

As before read the introduction, then pull down the help menu and select "concepts of ...". When you have finished reading pull down the suggestion menu.

1. Select suggestion 1. Follow instructions to locate the Earthquake. When you are successful record the coordinates of the Earthquake epicenter and the average time of the Earthquake's occurrence here.

Answer question a.

Answer question b

2. Select suggestion 6. Record the coordinates and average time of the Earthquake here.

Answer question a.

Answer question b.

When you have finished select QUIT from the file menu and let the next group have their turn. Once all of the students have completed the assignment you may, if you like, explore any of the programs on the diskette. This software is public domain so you may make a copy on your own diskette.

Chapter 6: Some Consequences of Plate Kinematics Unlocked by a Lava Lake Analogy

To an intelligent outsider the most striking geologic feature of the Earth would undoubtedly be the ~3 km high mid-ocean ridge mountain chains that encircle the Earth twice. We don't appreciate the magnitude and continuity of this mountain chain because its peaks lie beneath 2½ km of ocean water and the mid-ocean mountains are not steep-sloped. Like the great volcano of Hawaii, Mauna Loa, they have gentle slopes. However, if the oceans were stripped off, the amazingly continuous oceanic mountain chain would surely be the first feature any student of the Earth would feel compelled to explain. Anyone who explains this feature knows a good deal about the Earth and how it works.

When confronted with a problem in geology, a good natural analogy can be invaluable. What we know about the Earth to this point suggests it may behave a bit like a lava lake. The top surface of a lava lake cools quickly and forms a denser crust. In fact, the crust of the lava lake on Mauna Loa divides itself into plates. Where the plates spread apart, there are bright bands of glowing lava, particularly evident at night. The spreading is balanced by subduction, or underturning, of the lava crust, which is heavier than the hot material beneath because it is cooler.

Suppose the Earth behaves like a lava lake. Suppose the lithosphere (the cold, rigid outer shell of the Earth) forms at mid-ocean ridges and then moves away from the ridge, cools, thermally contracts, and thus becomes denser. The cold layer will become progressively thicker with time as it moves away from the ridges, like ice growing on a pond. If the crust is in floating equilibrium with the fluid mantle, the lithosphere will ride progressively lower as it thickens, much as barnacles growing on the keel of a boat might pull it down.

Could the thermal contraction of the outer surface of the Earth, together with Isostasy, account for the elevation of the mid-ocean ridges? This question is addressed mathematically on the following pages. The analysis shows that the lithosphere grows to ~ 100 km thickness in the ~100 million years it takes to open the Atlantic. The ~ 666K average cooling causes a thermal contraction of about 2.2 km (2.2%). Isostasy increases the subsidence by about 43%. This indeed accounts for the observed 3.5 km decrease in sea floor elevation from the middle to the edge of the Atlantic. The increase in the depth of the sea floor is proportional to the square root of the age of the ocean plate, and is the same for all ocean plates. This square root relation explains the depth of all oceans with remarkable precision. Plate tectonics, heat balance, and even a rough knowledge of the coefficient of thermal expansion, density, heat capacity, and thermal conductivity or rock material, plus some classical physics, are sufficient tools to explain the most important feature of our planet - the mid-ocean ridges that encircle the Earth twice.

6.1 Lava Lake Analogy to Plate Tectonics

The purpose of this section is to quantitatively understand how the Earth's plates form, their thickness, and their elevation. Using Fourier's Law of heat conduction, heat balance, and Isostasy (floating equilibrium) we will use the lava lake analogy to describe plate tectonics.

As illustrated in the figure on the next page, hot asthenosphere and magma wells up at ridges to fill the gap left by plates spreading apart, just as magma wells up in cracks opened by movements of the solid skin on a lava lake (Figure 6.1). As the new crust moves away from the ridge (or crack) it continues to loose heat to the atmosphere, and the thickness of the plate grows. At any time the temperature profile (Figure 6.2) through the plate is roughly linear to a certain depth, and then roughly isothermal (constant temperature), as shown in the linear approximation curve. We can use this linear approximation to estimate how the Earth cools and how the lithosphere (the cooled, relatively rigid, outer shell of the Earth) grows with time and distance away from a spreading ridge axis.

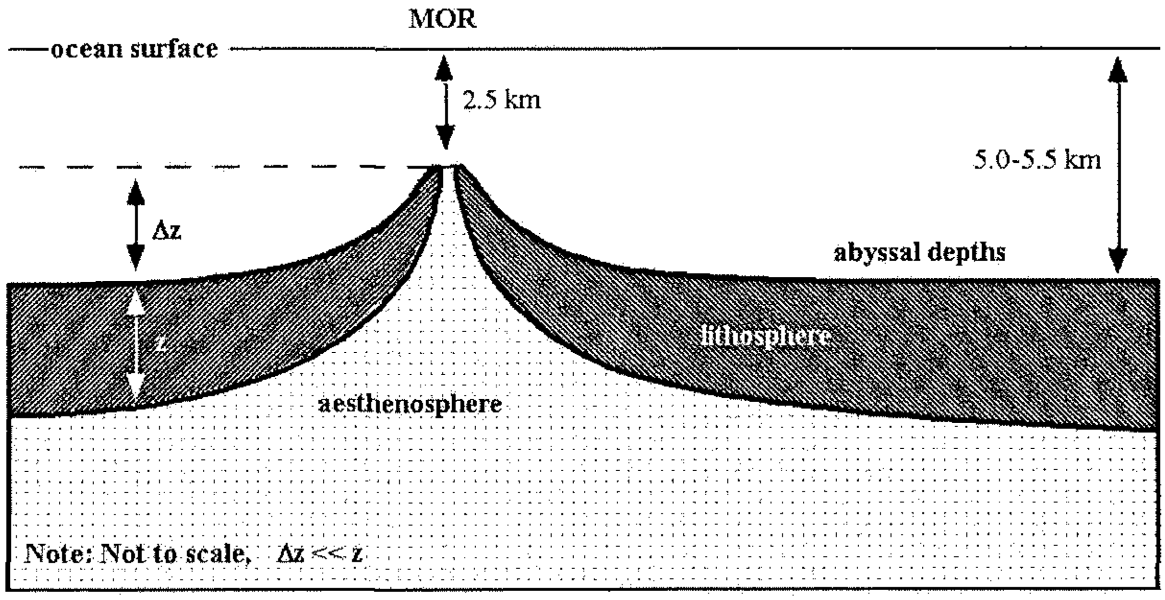


Figure 6.1

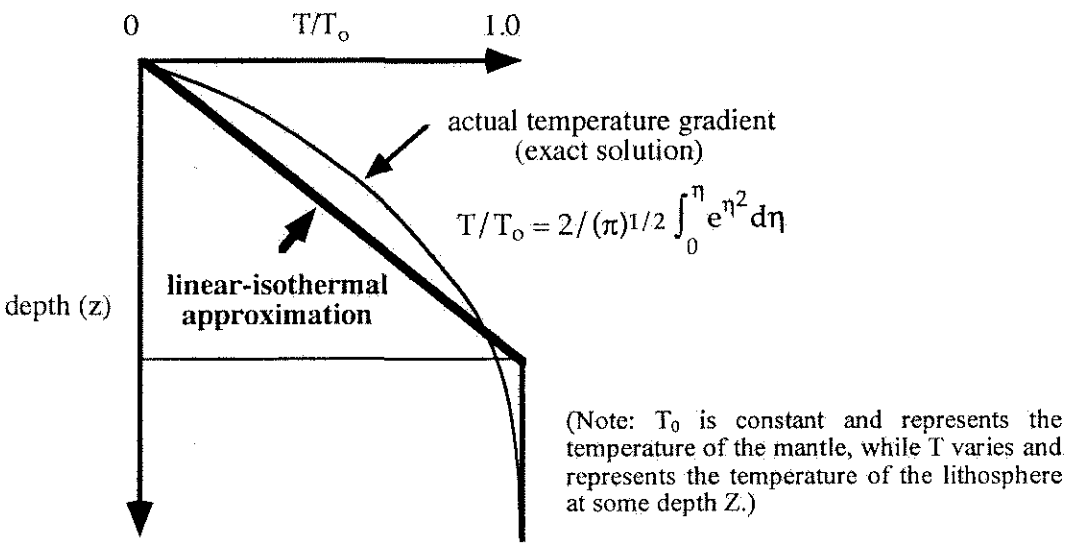


Figure 6.2 Vertical temperature profile through the Earth at some distance from the spreading ridge axis.

Fourier's Law states that heat is conducted down a temperature gradient at a rate proportional to the thermal conductivity of the medium in question (in this case mafic rock). Mathematically we may express this as:

$$\vec{J} = \frac{Q}{A \cdot \Delta t} = -K\frac{\partial T}{\partial z}\hat{z}$$

Where $\vec{J}$ is the heat flux in joules (Q) per square meter (A) per second (Δt), K is the thermal conductivity (a parameter which describes the ability of a material to conduct heat) in J/m-sec-K, and T is temperature in K. The minus sign simply means that if T increases as z increases, $\vec{J}$ will flow in the opposite direction (i.e. from the bottom of the lithosphere upwards).

In order to complete the calculation, we need to find a second expression, which describes the same thermal phenomenon. The thermal conduction described above is measured by the subsurface temperature gradient. The same heat flux can be described as the heat released when the lithosphere cools. The heat released by cooling is measured by the average change in subsurface temperature and the volume of mantle whose temperature is so changed. To estimate the cooling, we notice that at the stage illustrated in the temperature profile in Figure 6.2, the heat lost is $\sim\rho c T_o z/2$ joules per m^2 cross-sectional prism. Here ρ is the density of the rock in Mg/m^3, c is the heat capacity of the rock in J/g-°C, T_o is the initial temperature of the magma (rock in its fluid state), and z is the depth to the bottom of the cooled layer.

We can now equate the two expressions. Roughly, the temperature gradient at the base is zero, so by Fourier's law, no heat is conducted across the bottom boundary of the cooled zone (lithosphere). The temperature gradient (slope of graph) at the present time is about T_o/z, but in the past, when z was less, the gradient was greater. On average over the full time to cool to the present depth z, this gradient was twice the present value or $2T_o/z$.

Thus, if the time to cool to the depth z is t seconds, heat balance (i.e., the requirement that the heat lost by conduction equal that given off by cooling) mandates:

Integrated heat flux from Earth = heat lost by cooling

$$tK2\frac{T_0}{z} = \rho c T_0 \frac{z}{2}$$

(verify dimensional analysis agrees)

or

(1) $z = 2\sqrt{\kappa t}$

where $\kappa = K/\rho c$ is the thermal diffusivity in m^2/sec. The exact solution is compared to the linear approximation in the Figure 6.2. As you can see, the linear approximation is quite good. You should note carefully the logic of this solution. You can keep track of the money in your bank account by knowing the amount of money in it at some starting date and the amount that has been withdrawn since that date, or by integrating your average daily rate of withdrawal over the same period. The two should be equal. This is how we have solved the heat flow equation. We have estimated the total heat withdrawal and set it equal to an estimate of the integrated rate of withdrawal.

A number of consequences follow from the solution we have just obtained (equation 1): First of all, taking reasonable values of $K = 3.1$ W/m-K, $\rho = 3.3$ Mg/m^3, $c = 1.17$ J/g-K, then $\kappa = 8\times10^{-7}$ m^2/sec, and a 100 my old plate will be about 100 km thick. Secondly, we can estimate heat flow as a function of time using Fourier's law.

$$\vec{J}_{surface} = -K\frac{\partial T}{\partial z}\bigg|_{z=0} = K\frac{T_0}{z} = \frac{KT_0}{2\sqrt{\kappa t}}$$

Using the exact (error function) solution we would get $KT_0/(\pi\kappa t)^{1/2}$, which is very nearly the same. Using the parameter values given above and T=1333°C for the temperature of the uppermost mantle, we find:

$$\vec{J}_{surface} = \frac{0.47\,W/m^2}{\sqrt{t}}$$

where t is in [Ma].

We can now repeat Lord Kelvin's famous calculation of the age of the Earth based on its present heat flow. If the Earth were only a million years old, according to the above expression it would have a heat flow of 470mW/m^2. Presently the heat flow from the Earth is on average about 70mW/m^2. This suggests that the Earth is about 45 Ma old:

$$70\frac{mW}{m^2} = \frac{470\,mW/m^2}{\sqrt{t[\text{Ma}]}}$$

$$t = \left(\frac{470}{70}\right)^2 = 45\,\text{Ma}$$

It was the agreement of this (and other estimates) of the Earth's age with his similar estimate that 45 million years was about the time required for the sun to cool from its accretional temperature to a temperature consistent with its present luminosity that made Lord Kelvin absolutely certain that the Earth could not be older than a few 10's of millions of years. He was exasperated by the ignorance of the geologists of the day who would not abandon their notion that the Earth must be much older. He was not very polite and was quite wrong as it turned out because he, like Sir Harold Jefferies, had overlooked an important physical phenomenon. Can you think what it might have been?

Finally, we can calculate the topography of the ocean ridges from thermal contraction. Thermal contraction of the cooled part of the Earth's lithosphere can be expressed:

$$\Delta z = \alpha z \Delta \bar{T} = \alpha\left(2\sqrt{\kappa t}\right)\frac{T_0}{2} = \alpha T_0\sqrt{\kappa t},$$

where α is the coefficient of thermal expansion (measurement of how much a material expands when temperature increases) in °C^{-1}. Note that under the linear approximation, the average temperature of depth z is $T_0/2$. Taking $\alpha = 3.3 \times 10^{-5}$ K^{-1}, we can construct the table shown on the next page. In the last column, the change in elevation has been increased by a factor that takes into account isostatic equilibrium, or the loading of the sea floor with water as thermal contraction and subsidence take place. We will discuss how this is calculated in the next chapter. The figure below the table shows that when this isostatic factor is taken into account, there is almost perfect agreement between the depth of the ocean floor we would predict by thermal contraction and the depth observed until the ocean lithosphere is older than 150 Ma. After about 150 Ma the lithosphere appears to stop thickening and there is little or no additional thermal contraction and sea floor subsidence. Can you think why this might occur?

The relation between ocean depth and crustal age was noticed soon after the mid-ocean ridges were discovered during the 1958 Geophysical year. The rule that young ocean floor is shallow and old deep, and that the depth of the oceanic floor reflected its age became known as Hess' law. The previous equation is simply a quantification of this law.

We have been remarkably successful in understanding the origins of the most important mountain chain on the Earth – the mid-ocean ridges. We can quantitatively predict its form - the truest litmus of understanding.

t [Ma]	Z [km]	ΔZ [m]	ΔZ ($\rho_m/\rho_m - \rho_w$) [m]
1	10	220	316
10	32	698	1001
25	50	1104	1584
100	100	2209	3168
200	141	3122	4480

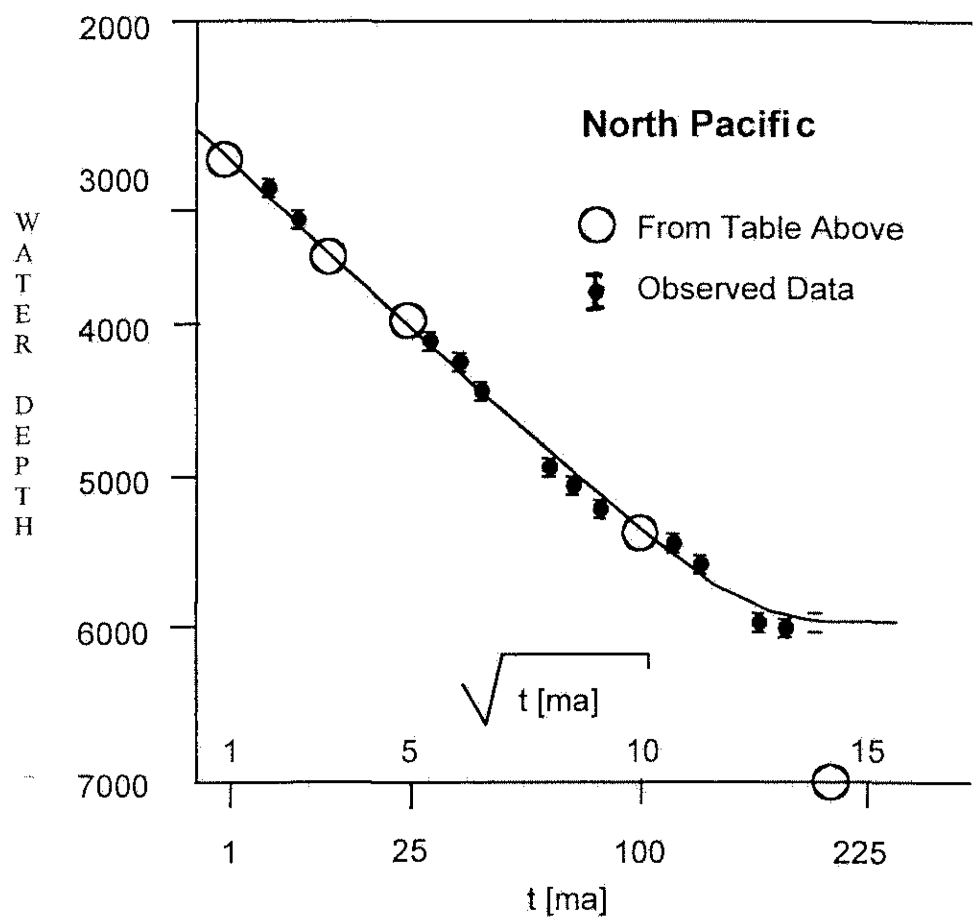


Figure 6.3. Elevation of the oceanic crust as a function of its age in Ma.

Problem Set 6a: Combining Thermal Contraction and Sea Floor Spreading

1. Calculate Z and ΔZ in the table on p. 30 above for $t = 10$ and 25 million years. Do you get the same results as the table? Define Z and ΔZ.

2.
   a. Write an expression for the thickness of the cooled part of the oceanic plate as a function of time.
   b. Show your expression is dimensionally correct.
   c. Assuming there is no water in the oceans, write an expression for the difference in elevation between two parts of the ocean where the crust is of different age.
   d. Does it make a difference if the two locations you have chosen are on the same plate? Why? If one mid-ocean ridge lies at 2.5 km water depth, will all mid-ocean ridges have the same depth? Why?
   e. Find the difference in ocean depth between a location where the age of the plate is 30 and 60 Million years old.

3.
   a. Use the expression in 2 to plot (on the same diagram) ocean depth as a function of distance from the ridge axis for ridges spreading at 10 mm/yr and 50 mm/yr half spreading rate (mm/yr is a useful unit because 1 mm/yr = 1 km/My). Assume no sediments cover the ocean floor.
   b. If these two spreading rates are typical of the Atlantic mid-ocean ridge and the East Pacific Rise, respectively, which has the steeper slope? Do you think this might explain why one is called a ridge and the other a rise? (the two different descriptive terms predated Plate Tectonics)
   c. If the average spreading rate of all ocean ridges were to increase, what would eventually happen to the average depth of the world's oceans? What would this cause the shorelines to do on the continents of the Earth? About how long would this take to happen?

4. If the diffusivity of molten magma has the same value we have used for the mantle, what will be the thickness of the thermal boundary layer on a lava lake after one hour? If the temperature of the magma is 1200°C, and the coefficient of thermal expansion is the same as the mantle, what is the elevation of the spreading ridge on the lava lake compared to the elevation of a location on the lake where the crust is 1 hour old?

5. Extra Credit:
   Calculate how fast ice will form on a lake in winter. Freezing of water is slowed by the latent heat of freezing as well as the heat capacity of water. Where in the heat balance equation on p. 28 could you add the latent heat of crystallization, L[J/g]? Derive an expression similar to $z = 2\sqrt{\kappa t}$ that includes L. The expression you obtain should contain L in the denominator of the modified equation, because the larger L the smaller the depth of cooling, z, at any time t. Furthermore the expression should reduce to $z = 2\sqrt{\kappa t}$ when $L = 0$. Your expression should be dimensionally correct. If ice has a thermal diffusivity of 10^{-6} m^2/s, a heat capacity of 2.09 J/g-°C, $L = 167$ J/g and the temperature at $t = 0$ drops suddenly form just above freezing to -10°C, how long will the pond take to develop an ice thickness of 10 cm?

6.2 Isostasy

A final piece of the puzzle we must understand is the puzzle of floating ice-berg equilibrium or Isostasy. If we place ice in water, 90% of the mass will sink and only 10% will project above the water surface. The density of ice is 0.9. In order to float the ice must displace exactly its own mass in water. This is the law of Isostasy or floating equilibrium. A similar phenomena will pertain if we load the Earth, because the Earth is a fluid. If we fill a 5.5 km deep ocean with sediments, the floor of the ocean will subside under the load until the sediments displace a mass of mantle material exactly equal to their own mass. By the time we fill the hole produced by the subsidence (and all subsequent subsidence due to the sediment loads) we find we must deposit about 16 km of sediments to fill the 5.5 km ocean. Similarly when the lithosphere is heated (or newly created) at ocean ridges it is thermally expanded and displaces a water load. As the plate cools and contracts the water load is increased. If the plate is in floating equilibrium with the underlying mantle, this loading will lead to an additional subsidence above and beyond the thermal contraction. The subsidence will be just such that the mass of mantle displaced below the plate equals the mass of water added above the plate. Living on a fluid Earth has important consequences that every student of the Earth should understand.

The dictum of floating equilibrium or Isostasy is a simple one: All constant cross section columns of material through a fluid down to some depth of compensation where there are no density irregularities must contain equal mass. For this to be so, an incremental mass floating on a fluid must displace an equal mass of fluid. The effects of Isostasy can thus be calculated in two ways: 1) the mass added equals the mass displaced, or 2) the masses within columns to some depth of compensation are equal.

Method 1 (mass-added-equals-mass-displaced)

Consider first the ocean ridge (Figure 6.4). If the thermal contraction is ΔH_{therm}, the water load (per cm^2 of surface) added by thermal contraction is $\rho_w \Delta H_{therm}$, and the isostatic subsidence, ε_I, is $\rho_w \Delta H_{therm} / \rho_m$ (see calculation in next page).

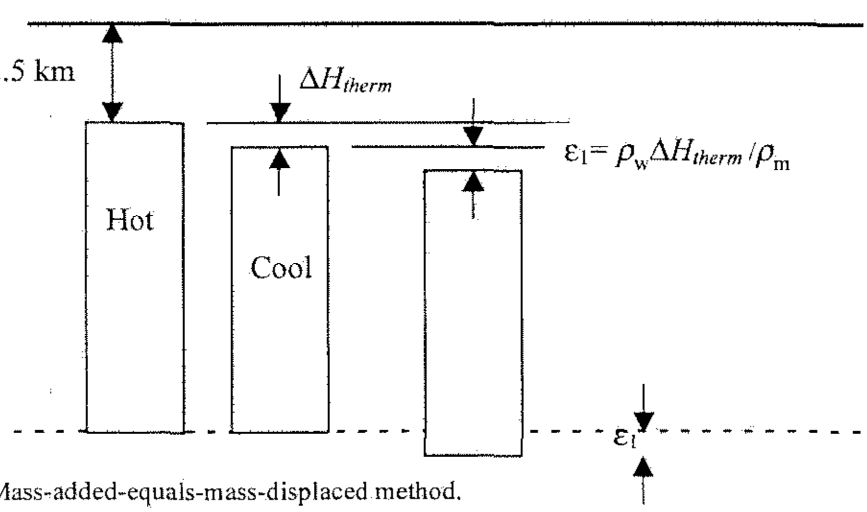


Figure 6.4. Mass-added-equals-mass-displaced method.

This isostatic subsidence is filled with water and this water load causes an additional subsidence, $\rho_w^2 \Delta H_{therm} / \rho_m^2$, which leads to a further water loading and further subsidence, $\rho_w^3 \Delta H_{therm} / \rho_m^3$, etc.

The initial isostatic subsidence, ε_1, can be calculated from the requirement that the mass of added water equal the mass of mantle displaced:

The mass of the water load added by thermal contraction = $\rho_w \Delta H_{therm}$

The mass of displaced mantle = $\rho_m \varepsilon_1$

Thus $\varepsilon_1 = \rho_w \Delta H_{therm} / \rho_m$

As suggested in Figure 6.4, the depression is filled with seawater, and this loading causes a second isostatic subsidence. This second subsidence is filled with seawater and produces a third subsidence, etc. The total ultimate subsidence is thus an infinite sum:

$$\Delta H_{total} = \rho_w \Delta H_{therm} \,[1 + \rho_w/\rho_m + (\rho_w/\rho_m)^2 + (\rho_w/\rho_m)^3 + (\rho_w/\rho_m)^4 + \ldots].$$

The sum to infinity of a geometric series such as that in parentheses is $\frac{1}{1-r}$, where r is the ratio of successive terms. Thus:

$$\Delta H_{total} = \Delta H_{therm}\left(\frac{\rho_m}{\rho_m - \rho_w}\right).$$

Method 2 (equal-sections-contain-equal-mass)

This same result can be obtained by going back to the basic definition of Isostasy: that equal sections through the Earth contain equal mass. Consider the two columnar sections as illustrated below:

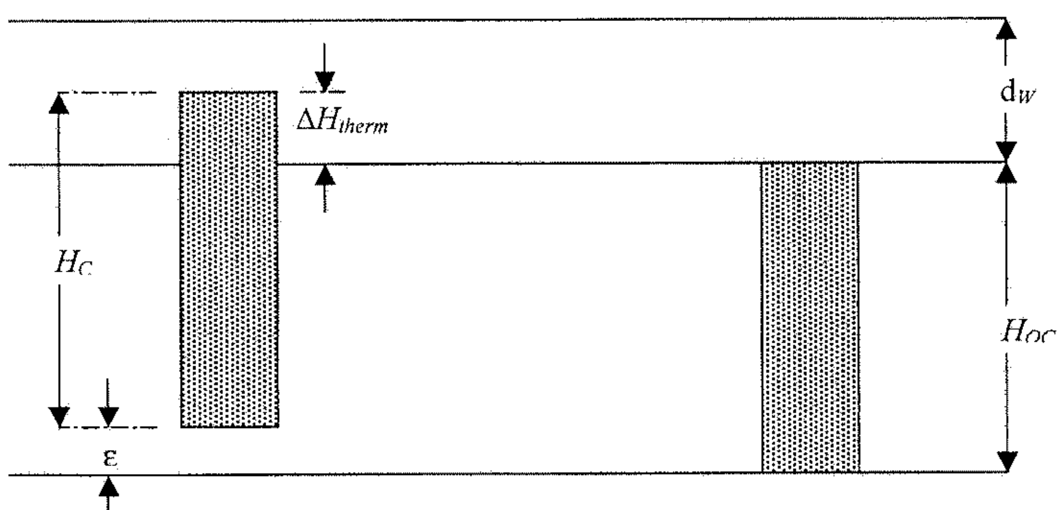


Figure 6.5

The sections containing the normal and thermally expanded crust must contain equal mass. We choose a depth of "compensation" as shown, define the thermal change in plate thickness $\Delta H_{therm} = H_C - H_{OC}$, and write the Isostasy requirement by setting the mass in the right column equal to the mass in the left:

$$\rho_w\, d_W + \rho_{OC} H_{OC} = \rho_w\,(d_W - \Delta H_{therm} - \varepsilon) + \rho_C H_C + \rho_m \varepsilon.$$

The second and fourth terms cancel because the "plate" contains the same mass despite its thermal expansion. The first term and the first component of the third term also cancel. Rearranging terms we see $\varepsilon = \rho_w \Delta H_{therm}/(\rho_m - \rho_w)$, and thus:

$$\Delta H_{total} = \Delta H_{therm} + \varepsilon = \Delta H_{therm}\left(\frac{\rho_m}{\rho_m - \rho_w}\right),$$

the same as above found by Method 1.

Over the long term, rock material behaves like sediment. Mountains are eroded, and the erosion products are deposited in depressions. If we adopt the terminology that a subscript "a" indicates thermal subsidence in air (e.g., the original topographic depression), "w" indicates the depth of the same depression if it is filled with water, "s" the depth of the depression if it is filled with sediments, and "c" if it is filled with crust then:

$$\Delta H_w = \frac{\rho_m}{\rho_m - \rho_w}\Delta H_a = 1.46\,\Delta H_a$$

$$\Delta H_s = \frac{\rho_m}{\rho_m - \rho_s}\Delta H_a = 4.73\,\Delta H_a$$

$$\Delta H_c = \frac{\rho_m}{\rho_m - \rho_c}\Delta H_a = 8.57\,\Delta H_a$$

The last expression is the same as the second except we use the density of continental crust (ρ_c) instead of sediments (ρ_s). The first two expressions can be combined to predict the thickness of sediments that could accumulate in an ocean of depth ΔH_w if it were filled with sediments:

$$\Delta H_s = \frac{\rho_m - \rho_w}{\rho_m - \rho s}\Delta H_w = 3.23\,\Delta H_w$$

In the above equations we have assumed $\rho_w = 1.0$, $\rho_s = 2.5$, $\rho_c = 2.8$ and $\rho_m = 3.17$ ton/m^3. The last expression indicates that if an ocean 5 km deep is filled with sediments, the total sediment pile will be 16.2 km thick. The second to last expression indicates that if a plateau originally 5 km high is eroded to sea level, the crust will be excavated to a depth of 42.8 km. Why is it appropriate to use ρ_c rather than ρ_s in the case of a plateau?

Examples of Isostasy

As an example of the **equal-sections-contain-equal-mass** method (Figure 6.6), consider ice floating in water:

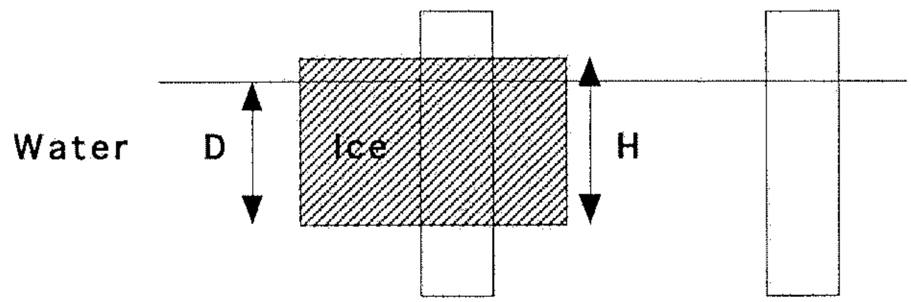


Figure 6.6

Clearly if $\rho_{water}D = \rho_{ice}H$, there will be equal masses in the two test columns to the depth of compensation. This means that

$$D = \frac{\rho_{ice}}{\rho_{water}} H = 0.9\, H$$

An iceberg floats about 90% under water because ice has a density of 0.9 and water 1.0.

As an example of the **mass-added-equals-mass-displaced** method, consider ice loading a continent (Figure 6.7). The continent and its lithosphere will be depressed under the ice load until a mass of mantle per unit area is displaced that equals the mass per unit of area of the glacial load. If the ice is 3 km thick, the isostatic subsidence will be about 940m. If the surface of the continent were close to sea level before glaciation, the surface would be depressed 940m below sea level after glaciation, while the top of the ice is 2 km above sea level.

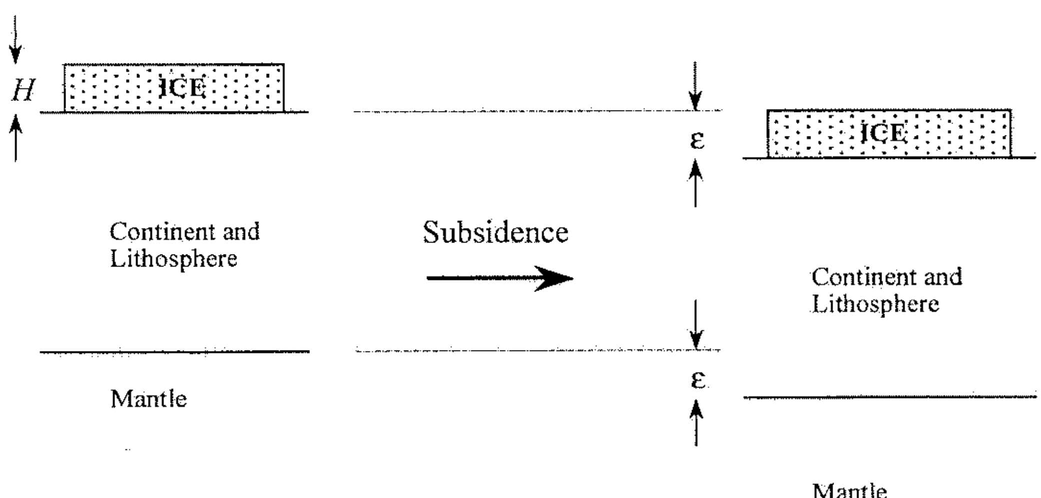


Figure 6.7

The subsidence is:

$$\varepsilon = \frac{\rho_{ice}}{\rho_{mantle}} H = \frac{1}{3.17} H$$

As a second example of the **mass-added-equals-mass-displaced** method, consider a continental plateau or mountain belt. If a layer of sialic (e.g., silica-rich compared to the mantle) continental crust with density 2.8 Mg/m^3 is placed on a continent, the layer will subside nearly its entire thickness.

$$\varepsilon = \frac{\rho_{continent}}{\rho_{mantle}} H = 0.88\,H$$

For example, if a 5 km sialic layer is added, after subsidence it will stand only ~600 m above the surrounding plains where a silica layer was not added. To produce a Tibetan Plateau with an elevation of 5 km above the Asian continent, a sialic layer ~42 km thick is required. In other words, mountains and plateaus must have thick buoyant (low density) roots if they are to be in isostatic equilibrium with their lower elevation surroundings. This explains why the plumb bob was not deflected by the Himalayas as expected. The buoyant, low-density root required to support the Himalayan edifice in floating equilibrium, canceled much of the attractive positive mass of the Himalayas. The attraction of the plumb bob was to a gravitational dipole (positive incremental mass in the edifice but negative incremental mass in the root).

The above result also shows how deeply a plateau can be eroded. When the Tibetan plateau is eroded to sea level, as it will ultimately be, the surface will consist of rocks that today are buried 42 km deep. Such erosion has occurred in the past, and this erosion has provided us with samples of rock that were once buried 42 km below the land surface. At great expense, the deepest humans have ever drilled is 12 km deep (Russian Kola Peninsula hole which took decades to drill). It would cost many billions of dollars to drill to 42 km depth. However, with understanding we can walk to localities that provide billions of tons of such material free. This is a good example of the cost effectiveness of understanding.

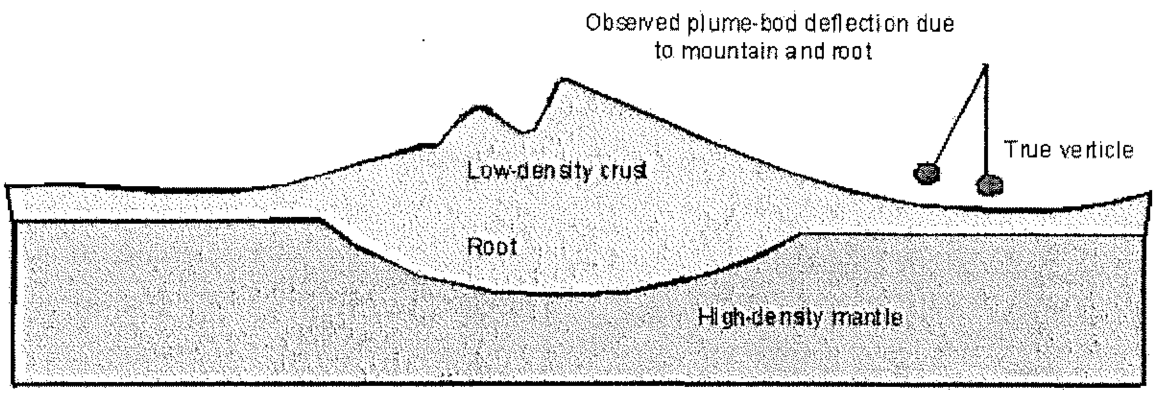


Figure 6.8 Plume bob experiment. (Adapted from Earth, p.492, 1986). Note crust-mantle root deflection is not drawn to scale.

Problem Set 6b: Thermal Cooling and Topography

1. As you determined in problem 2 of problem set 5, the difference in elevation of two ocean plate locations with age t_1 and t_2 is:

$$\Delta H_1 - \Delta H_2 = \alpha T_0 \kappa^{1/2} \left(\sqrt{t_1} - \sqrt{t_2} \right)$$

Modify your expression in problem 6a.1 to accommodate the case where the oceans contain water. What is the difference in ocean depth between points where the plate is 30 and 60 millions years old now? How much did filling the oceans with water rather than air change the elevation difference between the two ages of crust?

2. Where the mid-ocean ridge is offset by transform faults, plates of different ages are juxtaposed.
   a. Draw a cross section across a transform fault parallel to the ridge showing the change in elevation if the plates are 10 and 60 Ma.
   b. Use the ocean depths calculated in (1) to label the offset across the transform. Geologists call this kind of a step a "scarp".

3. Cold ocean floor lies 5.5 km below sea level.
   a. Fill the ocean with sediment with density 2.5 Mg/m^3, i.e., replace 1 Mg/m^3 sea water with 2.5 Mg/m^3 sediments or add a load 5.5 km high with density 1.5 Mg/m^3. How much will the ocean floor subside?
   b. Now fill the space provided by this subsidence with sediments and calculate the subsidence to this additional load.
   c. Keep doing this until the subsidence is very small. How thick is your sediment pile?

4. Determine the sediment thickness in (3). By applying the principal of Isostasy and requiring that equal mass be contained in prisms of equal cross section to some depth of compensation before and after filling the ocean completely with sediment. The discussion and example of Isostasy in the text should help you make both of these calculations. The two methods (3a, b, c) and (4) are equivalent. Did you get the same answer? Which do you find easier? Either method can be used to solve Isostasy problems. In the future, use whichever method you find easier.

5. Now consider erosion. If a broad plateau stands 2 km above sea level and is in isostatic equilibrium, how deep will erosion have penetrated when the plateau has eroded to sea level? Assume the density of the rock eroded is 2.7 Mg/m^3 and that Isostasy is maintained. Consider Isostasy only; assume that the thermal state of the plateau does not change. The answer is <u>not</u> 2 km. Why?

6. A mantle convection cell (plume) has been proposed to reset the thickness of the continental lithosphere from a thickness appropriate for an age of 115 Ma to a thickness appropriate for 25 Ma. It the continental crust in question was initially 0.5 km above sea level, what will be the elevation of the crust after the passage of the plume? Assume the continental plate can be treated as if it were an oceanic plate. Calculate the elevation of the plate under air (no water load) for the two ages of lithosphere.

Chapter 7: Earth Dynamics

In the early part of the 20th century (~1920's) attention was focused on the fluid nature of the Earth. Careful investigations in Fennoscandia (a term applied to the Scandinavian countries Sweden, Norway, Finland and Denmark) showed that the land was presently rising, and had risen hundreds of meters in a remarkably uniform domal pattern since post-glacial time. The center of the uplift dome is the north end of the Gulf of Bothnia (the sea between Finland and Sweden). The current rate of uplift there is about a meter every 100 years. Investigations of uplifted shorelines and sea level measurements in Canada showed the land surface was uplifting there as well, also in a domal pattern. This dome is centered on James Bay, and is rising at about the same maximum rate of ~1m/100 years. In fact, the land is uplifting in all areas glaciated during the Pleistocene - a phenomenon known as post-glacial uplift or postglacial rebound.

The rates are rapid enough to have been noticed by native people. Eskimos included the emergence of islands in children's stories about clams that led good lives and were elevated to the highest peaks (where clam fossils were observed in strata). Records of Hudson's explorations record entry into coves now, 300 years later, far too shallow to accommodate his vessel. Indian paintings made in winter on the shores of the great lakes are now far above human reach.

One significance of the ongoing glacial rebound is that it allows us to measure the viscosity (the property of a fluid to resist a force tending to cause the fluid to flow) of the Earth's mantle. (Equally significant when this was first noticed is that it was the first clear evidence that Earth's mantle behaved as a fluid on geologic time-scales). Nature performed a global experiment when the glacial ice melted about 10,000 years ago. Until about 12,000 years ago, extensive areas of the Northern Hemisphere were covered by continental-scale glaciers 1 1/2 miles thick. Starting ~12,000 years ago these glaciers melted rapidly. The last ice disappeared from Canada only about 5,500 years ago.

The way in which the glacial rebound is analyzed is typical of geophysical methods in general. The rate of rebound under small loads tells us the viscosity of the upper parts of the mantle. The rate of rebound of larger loads tells us about the viscosity of deeper parts of the mantle. By looking at various size loads, we can determine the viscosity of the mantle at all depths. The results are significant. Glacial rebound shows that the Earth's mantle has a viscosity low enough that if there is <u>any</u> temperature difference between the top and bottom of the mantle at all, the mantle must be vigorously convecting.

7.1 Earth's Rheology: Visco-elastic

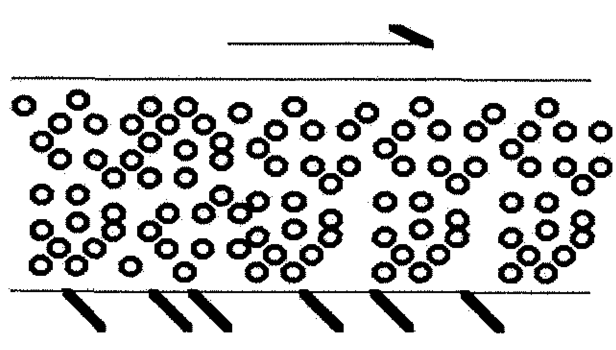


Figure 7.1 - The concept of rheology displayed by Bragg's bubble model.

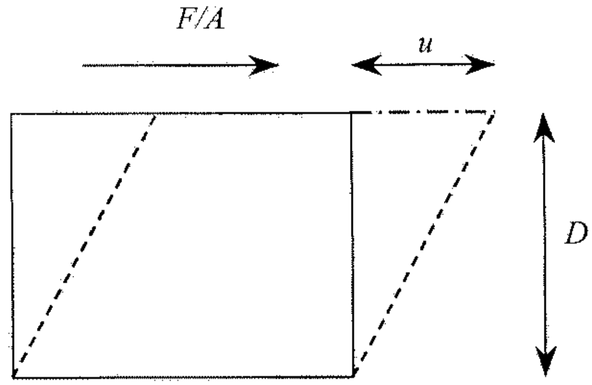


Figure 7.2 - Stress-strain relationship

Consider Bragg's bubble model (Figure 7.1). A layer of bubbles rests on a solid substrate and is subject to a shear stress on the top surface. When the stress is applied, the bubbles first elastically deform and then slide over one another. The elastic response is instantaneous; the viscous response occurs over time. Both are linear and can be superimposed (or added) to obtain the total response. If we define F as force in Newtons, A as area in m^2, u_e as the elastic displacement in m (Figure 7.2), u_v as the viscous displacement in cm, $\dot{u}_v$ as the rate of viscous displacement (or fluid velocity), and z to be the coordinate measuring depth into the bubble layer, the physical laws describing the deformation of Bragg's bubbles can be written

$$\mu \frac{\partial u_e}{\partial z} = \frac{F}{A},$$

and

$$\eta \frac{\partial \dot{u}_v}{\partial z} = \frac{F}{A}(t),$$

where:

$$u_e = \text{elastic displacement} = \frac{DF}{A}\left(\frac{1}{\mu}\right)$$

$$\dot{u}_v = \frac{\partial u_v}{\partial t} = \text{rate of viscous displacement} = \frac{DF}{A}\left(\frac{1}{\eta}\right)$$

In these expressions, μ is the elastic shear modulus of the bubbles in N/m^2 or Pa, and η is the viscosity in Pa-s. Since the elastic and viscous displacements add,

$$u = u_e + u_v$$

$$u = \frac{DF}{A}\left(\frac{1}{\mu} + \frac{t}{\eta}\right),$$

where D is the depth of the bubble layer.

In the Earth μ is about 10^{11} Pa, and η is about 10^{21} Pa-s. Thus, we see that if the viscous displacement is to be noticed, t must be comparable to η/μ or ~300 years. **This is the concept of rheology (the study of time-dependent material deformation and flow): material properties appear different depending on the time scale over which they are viewed.** Over short periods, the Earth appears to be an elastic solid. Over longer periods, it behaves as a viscous fluid. Silly putty is an excellent analog.

The total displacement equation also indicates scale is very important. For a given applied force, F, the magnitude of deformation is directly proportional to the depth of the bubble layer, D. If D is very large so is u! Thus, large bodies like the Earth behave very differently than human-scale objects (great differences also arise when going to smaller scales — e.g., small insect wings can 'use' air as a viscous fluid, unlike bird or airplane wings, while water's surface tension makes it a perfectly strong surface for waterbugs to walk upon). Even a very rigid body will behave as a fluid if it is large enough. Beware of intuition derived from everyday experience!

Determining The Earth's Viscosity.

The next issue is how to measure the Earth's viscosity. Its elastic properties are well measured by seismic techniques. The Earth's viscosity is best measured by glacial rebound. During the ice ages, glaciers covered Canada and the Scandinavian countries. The Earth came into isostatic equilibrium under the ice load and, when the ice melted, "rebounded" back into isostatic equilibrium.

Back-of-the-envelope calculation. We will first develop a simple back-of-the-envelope calculation to determine the viscosity of the mantle, and then show how a more complete solution can be developed that will yield a more accurate solution (and better justify the assumptions we needed to make in our simpler estimate.) Assume that the ice sheet has the form sketched in Figure 7.3a, extending over a width d that is typically hundreds to thousands of kilometers. Assume the iceload reaches isostatic equilibrium, depressing the land beneath it by an amount h_0. Once the load melts (assume this is instantaneous), the underlying mantle will rebound. Viscous flow will occur over a lengthscale L roughly equal to the width of the load, with a maximum upward velocity W at the top of the mantle beneath the load. (These assumed features of the rebound flow should be checked, if possible, against a more complete model...)

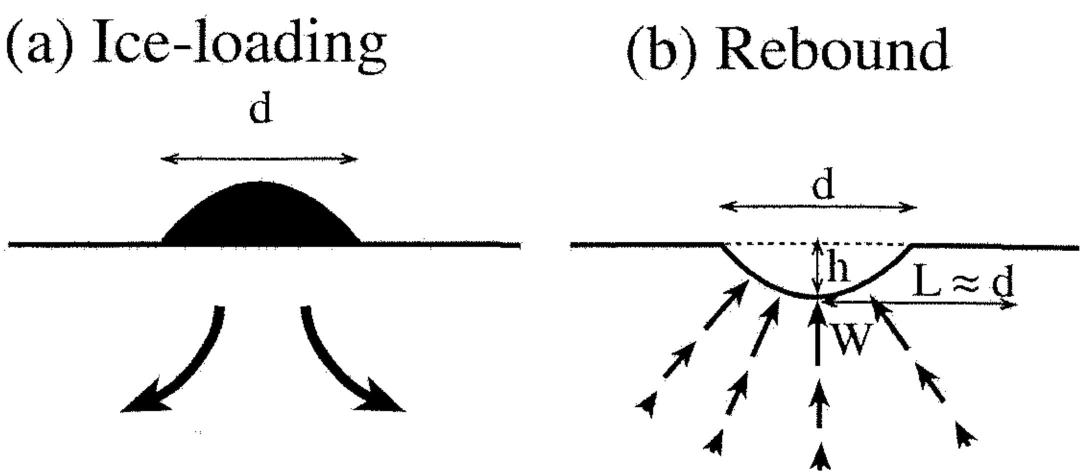


Figure 7.3 Sketch of (a) ice-loading and (b) rebound configurations assumed in a simple model to use rebound to estimate the viscosity of the mantle.

Now lets construct a simple force balance from this idealized picture. The upward restoring force of the 'hole' left by the melting ice is equal to $\Delta\rho g h = (\rho_{mantle} - \rho_{water}) g h$ (this assumes the depressed hole is filled up to sealevel by water). The viscous flow-induced stress that resists rebound is proportional to $\eta W/L \approx \eta W/d$. Now we can rewrite W in terms of the change in h with time: $W(t) = \dot{h}(t) = dh(t)/dt$. Equating the expressions for the rebound-driving and rebound-resisting forces, we find $\Delta\rho g h \propto \eta \dot{h}/d$ or $\frac{dh}{dt} \propto \left(\frac{\Delta\rho g d}{\eta}\right) h$. This equation has the same form as the previous one for radioactive decay, and has a solution in the form of a decaying exponential with time: $h(t) = h_0 \exp(-t/\tau)$, where $\tau = \eta/(\Delta\rho g d)$ is the exponential decay time. (By historical accident, half-lives have not been used to characterize rebound decay times.) If we now choose (see the table or rebound constraints following the next worked solution) an appropriate width for the Laurentian (North American) icesheet to be ~3000km, its decay time as measured by uplifted beach terraces in Hudson's bay to be ~2500yr, and the appropriate density contrast to be 2.3Mg/m^3, then we find that this analysis implies the mantle viscosity beneath North America to be $\eta \approx \Delta\rho g d \tau = (2300\, kg/m^3)(10\, m/s^2)(3x10^6 m)(2500x3.15x10^7 s) \approx 5x10^{21} Pa - s$. This estimate is not too bad, it has overestimated the mantle viscosity by about a factor of three. The model also clearly shows the controlling physics of rebound, and the exponential nature of rebound decay — the rate or rebound slows exponentially with time because the 'hole' or mass-deficit driving rebound back towards isostasy is continually decaying with continued rebound. Finally, the expression for the exponential decay-time shows the perhaps paradoxical feature that the rebound rate is faster under wide ice-loads than narrow ones. These are general characteristics of rebound flow. One could extend this approach to try analyze a case where the return flow is confined to a thin low-viscosity channel (in which case the characteristic lengthscale for flow may become the thickness of the channel instead of the width of the load). However, back-of-the-envelope calculations are usually best left simple — for more insight its usually more fruitful to try to develop a more accurate model. For viscous flow, an exact solution for a single Fourier harmonic of a load provides a powerful tool for determining the viscosity of the mantle.

Simple analytical model (an exact solution for a sinusoidal loading geometry). For linear viscosity the best way to proceed is to solve a simple problem and add the simple solutions to obtain results that are more complex. In this vein, consider the problem of the "isostatic adjustment" of a fluid whose surface is initially deformed in a sinusoidal pattern, as sketched below.

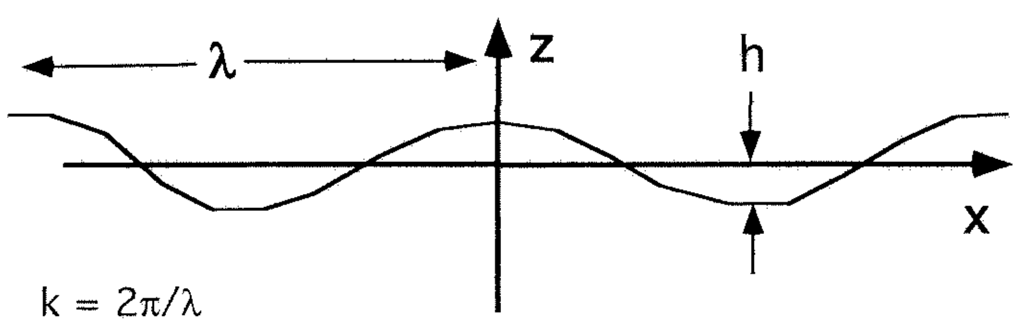


Figure 7.4

We seek a solution to the Stokes equation, which describes fluid flow in viscous material by requiring conservation of fluid momentum. For very viscous fluids such as we deal with here, the Stokes equation takes a particularly simple form:

$$\eta\nabla^2 v = \nabla p \,,$$

where

$$\nabla^2 = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right).$$

Again η is the viscosity of the fluid in Pa-s, p is pressure exerted on the fluid in Pa, and v is the fluid velocity vector. We define a boundary condition to be $p = \rho gh \cos kx$ when z = 0. Here g is gravitational acceleration in m/sec^2, z is depth (positive upward), k is the wave number defined in the sketch above and ρ is the density of the fluid (in this case, the mantle). Qualitatively, this means that when $z = 0$, the pressure is the weight of the fluid above or below the $z = 0$ horizon. The expression ρgh is the weight at some particular point which is also dependent on the sinusoidal deformation, hence the *cos kx* term.

The solutions to the Stokes equation subject to this boundary condition are given below:

$$p = \rho g h e^{kz} \cos kx$$

$$v_z = \frac{\rho g h}{2\eta k}(kz - 1)e^{kz} \cos kx$$

$$v_x = \frac{\rho g h}{2\eta k} kz\, e^{kz} \sin kx$$

You can see that at $z = 0$ the pressure is as proscribed. You can verify the validity of the solution by substituting into the Navier Stokes equation. Immediately after the load is applied, the surface has the vertical velocity indicated by the v_z equation. Since at the surface ($z = 0$),

$$v_z = \frac{\partial h}{\partial t} = -\left(\frac{\rho g h}{2\eta k}\cos kx\right) \text{ ,or solving for } h:$$

$$h = h_0\, e^{-\left(\frac{\rho g}{2\eta k}\right)t} = h_0\, e^{-t/\tau}$$

where $\tau = \dfrac{2\eta k}{\rho g}$ is the exponential decay time.

Notice that the decay time does not depend on the original amplitude of the load, but only on its wavelength k. A loaded surface will approach isostatic equilibrium exponentially in time regardless of the magnitude of the load (amplitude of mantle deformation).

The depth to which flow penetrates for a given wavelength load is easily estimated. A plot of v_z/v_z $(z = 0)$ shows that the vertical velocity drops to half its surface value above the depth where $kz = 1.66$. Half the flow occurs above this depth. Show that 80% of the flow occurs above $kz = 3.0$.

We now know how a sinusoidally loaded Earth will respond. Real loads are not sinusoidal. However, a load of arbitrary shape can always be constructed by superimposing sine and cosine loads of different wavelengths. A bar load, for example, can be synthesized by a relatively long wavelength cosine load that captures the central parts of the load, and higher frequency (shorter wavelength) loads that sharpen the edges so that they are square. The isostatic response of the central parts of the load with length L and width M can in fact be described well by an equivalent cosine with wave number k where:

$$k = 1.7\left(\frac{1}{L^2} + \frac{1}{M^2}\right)^{1/2}$$

So, for example, $k = \dfrac{1.2}{R_0}$ for a cylindrical square edged load of radius R_0, and $k = \dfrac{1.7}{L}$ for a long (very large M) narrow load of width L.

Using the above equations and wave number approximation, we can easily estimate the viscosity of the Earth's mantle from its isostatic response to the removal of loads of different scale and geometry. The decay time, τ, in years is inferred from the uplift history for each area. The scale of the uplift is determined from the uplift pattern. The calculated viscosity and the depth above which 80% of the flow will occur are fundamentally most significant, and are summarized in the table below:

Location	τ [yrs]	Scale [km]	η [Pa-s]	Depth - 80% [km]
Greenland	1500	$L = 60$	2×10^{19}	105
Bonneville	4000	$R_0 = 95$	1.6×10^{20}	237
Fennoscandia	4400	$R_0 = 550$	1×10^{21}	1375
Canada	2500	$R_0 = 1650$	1.7×10^{21}	4125
Oceans	<2000	$R_0 > 3000$	$<2.5 \times 10^{21}$	whole mantle

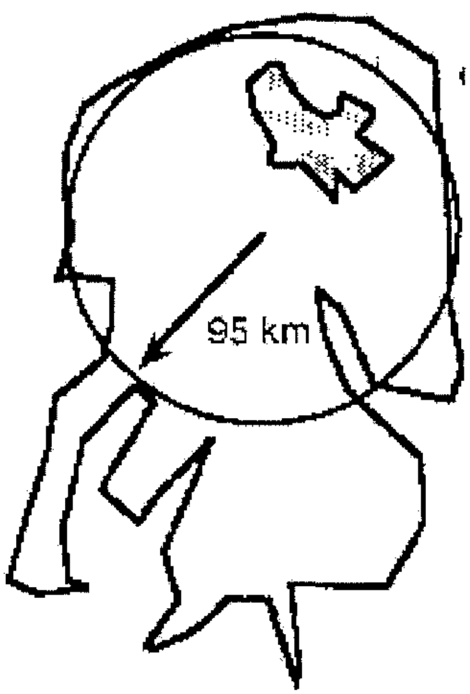
You can see that the Earth's mantle is of reasonably uniform viscosity (10^{21} Pa-s), except for the uppermost parts of the mantle (the asthenosphere), which is almost two orders of magnitude weaker. We show in the next section that this fluid mantle must vigorously convect. This vigorous convection has many geological consequences, which are observed, as we shall see. The viscosity of the mantle, which we determined using some simple physics and mathematics, tells us a lot about how the Earth operates.

Before turning to the problem set, it is worth noting that as in the Bragg Bubble model, similar solutions are obtained for the immediate elastic response. The elastic solution has the same form as the solution for slow viscous flow. As the following example illustrates, the maximum elastic response can be estimated from the size and magnitude of the load, similar to approximating the maximum initial surface depression rate. However, in the elastic case h refers to the height of the load and ρ refers to the density of the load. In the viscous case h refers to the isostatic deformation of the mantle and ρ refers to the density of the mantle.

As a first example, consider the classic example of the deformed shorelines of glacial Lake Bonneville, in Utah. Figure 7.5 shows the outlines of Lake Bonneville. In the upper right-hand corner is The Great Salt Lake. Lake Bonneville was much larger than the Great Salt Lake. When the Pleistocene (glacial) epoch ended about 12,000 years ago, Lake Bonneville dried up almost completely. (What's left is the Great Salt Lake we see today.) Lake Bonneville was almost 1000 ft deep (305 meters) and its shorelines are well incised, not only on the margins of the ancient lake but also on the islands that existed within it. A good example is Deer Island in the present Salt Lake, which can be easily observed from Salt Lake City. Many of the housing developments in the hills surrounding Salt Lake City are built on the flat incised shorelines of Lake Bonneville. The interesting thing about these shorelines, as documented beautifully by the famous Geological Survey geologist Lowell Gilbert, is that the shorelines in the central parts of ancient Lake Bonneville are elevated in a regular domal pattern about 65 meters relative to the shorelines at the edge of the ancient lake. The full isostatic response of the mantle to 305 m water load is 96.2 m, so the Earth reached within ~ 65% of isostatic equilibrium in the time that water filled Lake Bonneville, and has returned close to isostatic equilibrium in the time interval since the water evaporated. Crittenden showed that for 65 m of relative shoreline doming to be observed today, the response time of the mantle to the likely loading history of Lake Bonneville must have been 4000 years or less.

Using these observations, we can conclude that the mantle under Utah has an average viscosity of 1.65×10^{20} Pa-s to a depth of at least 237 km. Furthermore, we can show that the elastic response could at most be about 2 m. Thus, the elastic response cannot account for the observed doming.

Example of Lake Bonneville



Lake Bonneville Data

Max depth: 305m
Approx. radius load: 95 km
Observed uplift of central shorelines: 65m
Response time of uplift: ~ 4000 yrs

Maximum Isostatic Response

$$\frac{305\rho_w}{\rho_m} = \frac{305 \cdot 1}{3.17} = 96.2\,\text{m}$$

Equivalent Central Wave Number

$R_0 = 95$m
$k \sim 1.2/R_0 = 1.26 \times 10^{-9}\ \text{m}^{-1}$.

Figure 7.5

Expected elastic response:

$$u_e = \frac{\rho_w g h}{2\mu k} = \frac{\left(10^3\, kg/m^3\right)\left(10\, N/kg\right)\left(305 m\right)}{2\left(7\times10^{10}\,\text{N/m}^2\right)\left(1.26\times10^{-9}\ \text{m}^{-1}\right)} = 1.72\ \text{m}$$

(Note: we dropped the minus sign, understanding the elastic response is downward. Also, we use the value for μ determined by the seismic speed in this region, μ~7x10^{10} Pa.)

Viscosity for τ = 4000 years:

$$\left(4000\ \text{yrs}\right)\left(3.15\times10^7\,\text{sec/yr}\right) = \frac{2\eta k}{\rho_m g}$$

$$\eta = \frac{(4000 y)(3.15\times10^7\, s/y)\left(3.17x10^3\text{kg/m}^3\right)\left(10\, N/kg\right)}{2\left(1.26\times10^{-9} m^{-1}\right)} = 1.58\times10^{20}\left[\frac{\text{N-sec}}{\text{m}^2}\right] \text{ or Pa-s.}$$

Depth of flow

$$z_{50\%} k = 1.66$$

$$z_{50\%} = \frac{1.66}{1.26\times10^{-9}\ \text{m}^{-1}} = 131\ \text{km}$$

$$z_{80\%} = 237\ \text{km}$$

Problem Set 7: Viscosity of the Mantle

1. The maximum rate of vertical isostatic adjustment under a cosinusoidally deformed fluid surface occurs at $x = 0$, $z = 0$ and has a magnitude (as shown by the formula for v_z given on page 40 notes) at any time t of $v_{z_{max}} = \dfrac{\rho g h}{2\eta k}$.

   The same expression can be used to calculate the elastic response to a cosinusoidally distributed surface load if η is replaced by μ.

   a. What would h and ρ refer to in the elastic case?

   b. In the physical derivation of elastic and fluid displacement, it was shown that both elastic displacement and rate of viscous displacement are proportional to the scale of the flow as represented by D. Is $v_{z_{max}}$ also dependent on the size of the load? How?

   c. If two fluid surfaces are deformed cosinusoidally with equal amplitude, but one has twice the wavelength of the other ($\lambda_2 = 2\lambda_1$), how much more (or less) rapidly will the deformation with the larger amplitude adjust toward isostatic equilibrium? E.g., what will be the relationship between $v_{z_{max}}$ for λ_2 and $v_{z_{max}}$ for λ_1 at $t = 0$? Will the relationship be the same at later times?

   d. What will the relationship be between the decay times, τ, for the two deformations? Does the decay time depend on h?

2.

   a. Using the equations given on page 40, plot the vertical and horizontal flow beneath the cosinusoidally deformed top of the mantle to obtain a feeling for how the flow occurs. This can be done most easily by plotting the vertical velocity divided by $\dfrac{\rho g h}{2\eta k}$ (e.g., $v_z/v_{z_{max}}$) along the x-axis at $z = 0$ from $kx = 0$ to $kx = \pi$, and the horizontal velocity divided by $\dfrac{\rho g h}{2\eta k}$ along a vertical axis at $kx = \pi/2$ from $kz = 0$ to $kz = -\pi$. Remember that vertical is defined positive upward (see diagram p. 39).

   b. Sketch in the flow lines (e.g., the trajectories along which mantle material will move as isostatic adjustment occurs).

   c. Indicate with dashed lines the depth above which 50% and 80% of the flow occurs (using the expressions given on p. 43). Are these boundaries realistic in terms of the horizontal and vertical flow profiles you have sketched along the $kx = \pi/2$ and $kx = 0$ axes? Do your values of $v_z/\dfrac{\rho g h}{2\eta k}$ at $kz = 1.66$ and 3 coincide with those given in the notes?

   d. If two fluid surfaces are deformed cosinusoidally with equal amplitude but one has twice the wavelength of the other ($\lambda_2 = 2\lambda_1$), how much deeper will the flow persist in the second case? Explain how this feature can be used to determine the viscosity profile of the mantle by looking at the isostatic adjustment of loads of different sizes.

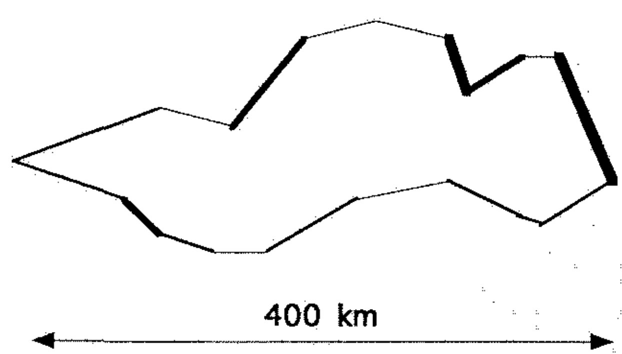
3. An ice load 5 km thick, 500 km wide, and 700 km long is suddenly deposited on a Cretaceous continent.

a. Find the wave number, k, which characterizes this load at the center of the loaded area.
b. Use this k to determine the maximum elastic depression of the surface that could be caused by the mass of ice. Assume $\mu = 7 \times 10^{10}$ N/m^2.
c. What will the depression under the ice load be once isostatic adjustment is fully attained, assuming the density of the mantle is 3.17ton/m^3 and the density of the ice is 0.9 ton/m^3?
d. Now show how long it will take to establish isostatic equilibrium under the load if the viscosity of the mantle to at least 1400 km depth is 10^{21} Pa-s. Do this by calculating the decay time for the stated load geometry and mantle viscosity and plotting the subsidence of the center of the loaded area versus time since the application of the load. You will find this easiest if you plot the logarithm of the remaining isostatic adjustment against time since the application of the load.

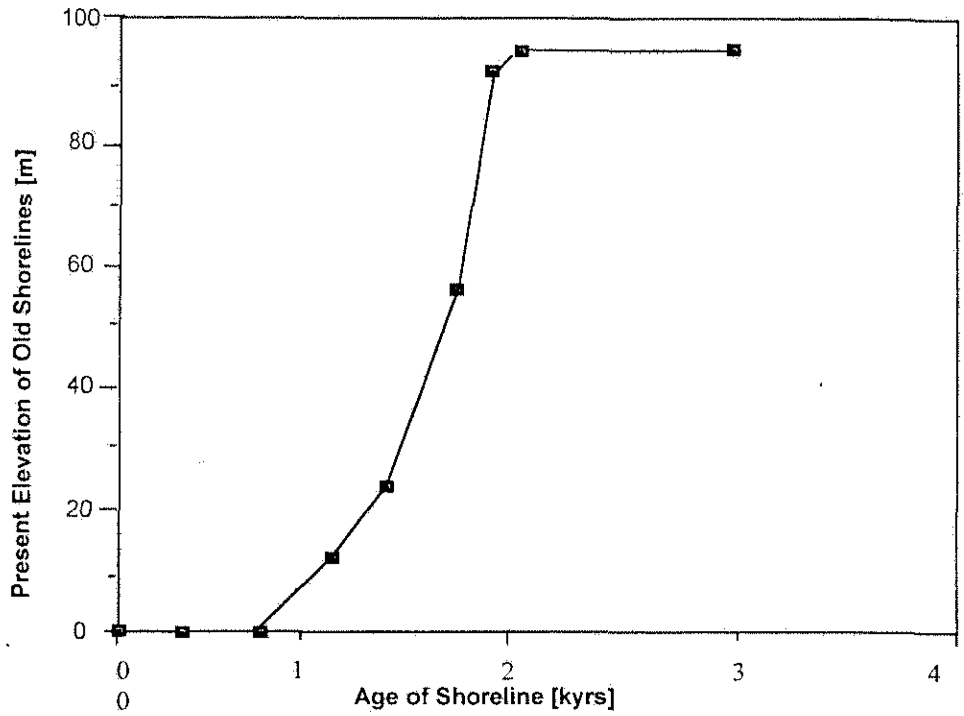
4.

a. Estimate the elastic response of the Earth under the center of the Fennoscandia Glacier (whose radius R is ~ 550 km). Assume the elastic shear modulus of the lithosphere and mantle, μ, is = 7×10^{10} N/m^2. Assume the ice is 2.7 km thick.
b. What is the elastic displacement under the load when the Earth has come into isostatic equilibrium under the load?
c. What percentage of the full isostatic adjustment is the initial elastic displacement?

5. Suppose Lake Superior were suddenly to drain. If the present water depth is 330 meters, how much would the central areas uplift? Estimate how long this would take to occur using your knowledge of the viscosity structure of the Earth.



6. Estimate the viscosity of the Earth's asthenosphere under Greenland if shoreline elevations at present have the following elevation-age relation:



Assume that the glacial retreat responsible for the uplift was a 100 km wide strip over 1000 km long. How deep will the mantle flow be in this loading case? How deep will the viscosity of the mantle affect the uplift response? How thick was the ice load before it melted about 10,000 years ago?

7. A circular depression 5.4 cm in diameter is impressed on the surface of a tub of silly putty 15cm deep. The circular depression is 66% erased in 5 minutes by fluid flow. What is the viscosity of the putty if the density of the putty is 2 g/cm^3?

7.2: Mantle Plumes and their Effect on Earth Topography

A dimensionless number called the Raleigh Number determines whether a fluid heated from below will convect or not. If the Raleigh Number is above 1000, convection will occur. If it is much greater than this value, very vigorous convection will occur. The Raleigh Number is the ratio of convective to diffusive heat transport. The critical Raleigh Number determines when the convective transport of heat is fast enough that thermal plumes can be maintained against diffusive heat losses so as to maintain steady convection.

The Raleigh Number is defined below:

$$R = \frac{\alpha \rho g d^3 \Delta T}{\kappa \eta}$$

where,

α = coefficient of thermal expansion	$= 2 \times 10^{-5}\ K^{-1}$
ρ =density of the fluid	$= \sim 4\ Mg/m^3$
g = gravitational acceleration	$= 10\ m/s^2$
κ = thermal diffusivity of fluid	$= 10^{-6}\ m^2/s$
η = viscosity of the mantle	$= 10^{21}$ Pa-s
ΔT = adiabatic temperature difference across layer	
R_c=1,000.	

Substituting the above values, which are appropriate for the Earth's mantle, we find that $R = 21{,}600\ \Delta T$, and $R/R_c = 21.6\ \Delta T$.

Thus, the mantle has a highly super-critical Raleigh Number even if the adiabatic temperature difference between the top and bottom of the mantle is only 1°C. The temperature difference is almost certainly much greater than this because the outer part of the Earth's iron core is vigorously convecting (to produce the Earth's magnetic field). Thus, the Earth's mantle is also vigorously convecting. Cylindrical mantle plumes connect the bottom and the asthenosphere (top of the mantle) as shown in the Figure 7.6. The asthenosphere is more fluid than the underlying mantle because it is hotter.

Mantle plumes, or buoyant masses of hot mantle material that rise to the base of the lithosphere, act like hot candles on the overlying lithosphere. They heat and thin the thermal boundary layer (lithosphere) as indicated by Figure 7.7, typically to the thickness of 25 Ma old oceanic lithosphere. Using the relations in Chapter 5 you should be able to show that resetting a 100 Ma ocean-covered lithosphere to 25 Ma will cause 1.6 km of uplift. Resetting 200 Ma ocean-covered lithosphere will produce an uplift of 2.9 km. A plume passing under a 200 Ma continental lithosphere will produce 2.0 km of uplift (assuming no erosion). Why is the continental uplift less than the oceanic? Could the continental uplift be a great deal greater if there is erosion?

Plume-like mantle convection explains most of the remaining topography of the oceans after the ridges have been accounted for by Isostasy. Chains of islands and guyots (islands that have been eroded to flat-topped atolls and then sunk) are plume tracks. The Hawaiian Island chain is an example. The big island of Hawaii is an active volcano because it sits right over a mantle plume. Yellowstone National Park, with its geysers, hot spring activity, and very recent volcanism sits over another mantle plume. Iceland sits over a third, etc. Plume tracks crisscross the oceans of the world. These tracks' position, which is relatively constant, records the absolute motion of the plates over the mantle. The magnetic lineations record the relative motion of the plates. The two are consistent and mutually confirming – at least to first order and the errors in measurements.

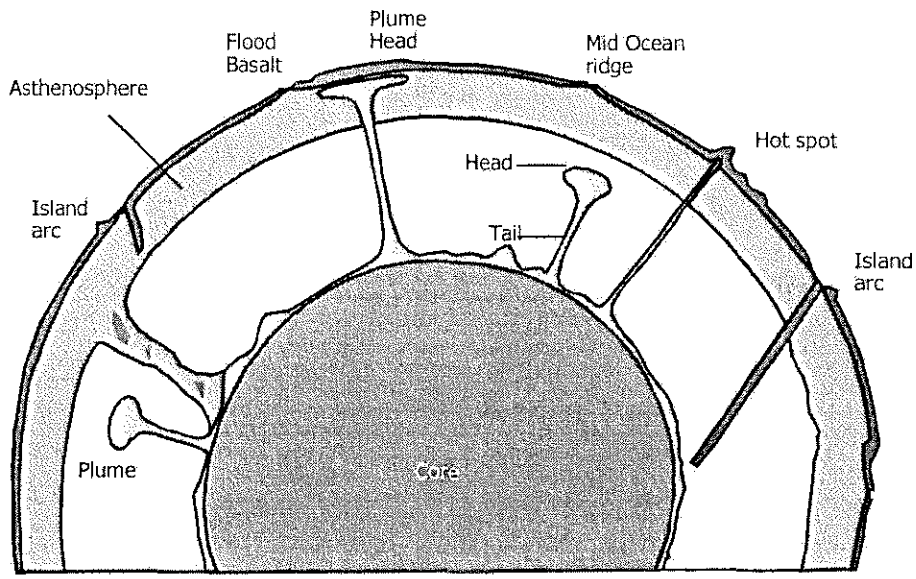


Figure 7.5 Schematic diagram of plumes rising from the core-mantle boundary. (Adapted from Earth's Dynamic Systems, p.591, 1997)

Plumes were the "cookie cutters" that sliced Pangea into such characteristic forms. The tracks weakened the lithosphere along eastern South America and western Africa, for example, producing the 90° coastlines that caught the observant eye of Sir Francis Bacon.

One further matter: mantle plumes can die and be born. When a new plume leaves the lower thermal boundary layer of the mantle, fluid dynamic experiments suggest it will have a large tadpole-type head (Figure 7.6). When the plume reaches the top of the mantle and this head impinges on the lithosphere, it has an unusually large thermal effect. Massive outpourings of basaltic magma are often the result. The Tristan plume produced the Serra Geral flood basalts and metamorphosed an area, which is today a famous mineral collecting and mineral resource area in Brazil. The Reunion Plume (now under Reunion Island in the Indian ocean) produced the Deccan Traps in India when it started up 66 million years ago. You will remember that this was the time of the extinction of the dinosaurs. Some believe that it was this plume-related outpouring that killed the reptiles, not a meteorite impact.

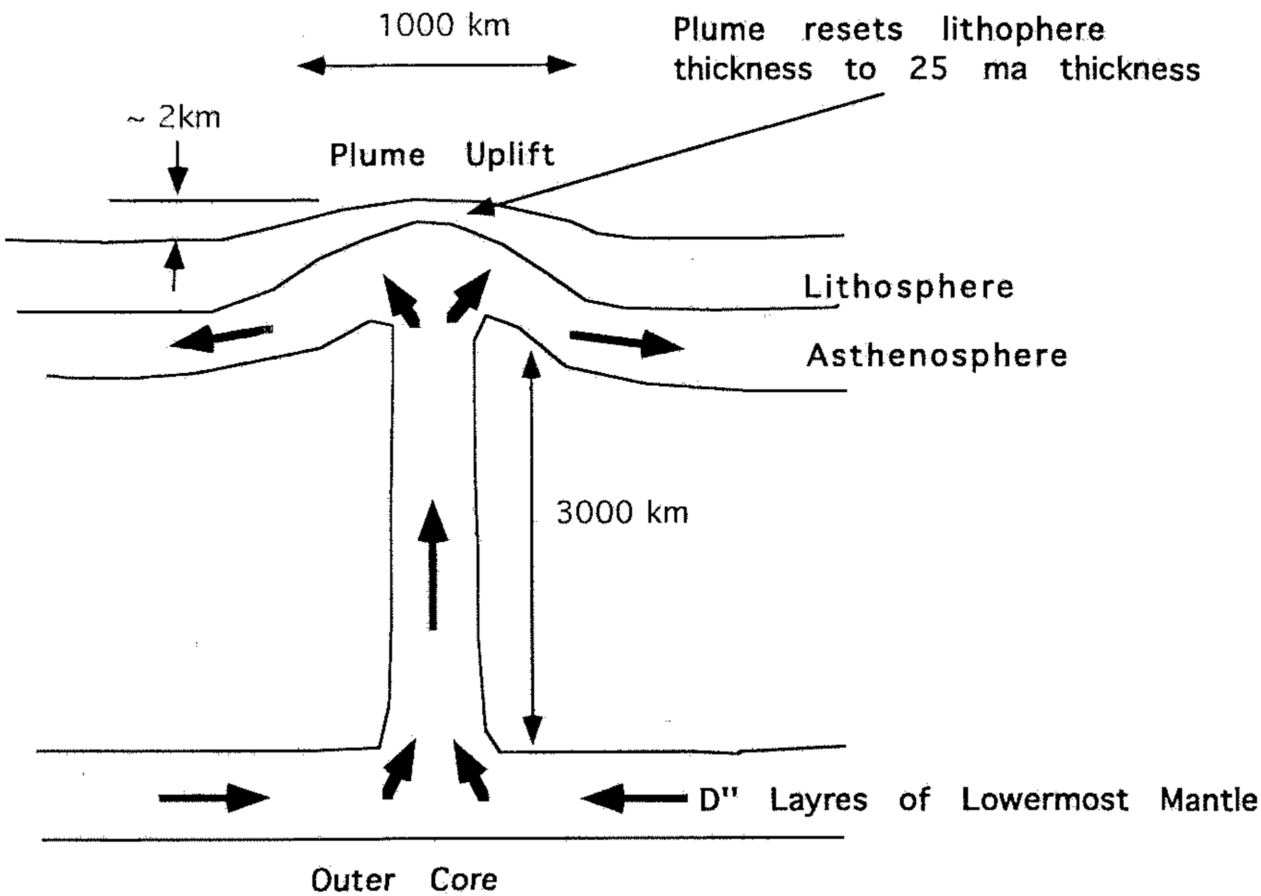


Figure 7.6 Mantle plumes thin the thermal boundary layer.
(D`` is the name of the mantle's bottom seismic layers)

Chapter 8: The Geology of North America

The geology of North America nicely illustrates the dynamics of the solid Earth. Consider the geology you would encounter on a trip through the U.S. from the east to West Coast and then north from Los Angeles to Oregon and Washington. In the east are the Appalachian Mountains, an old mountain chain that has largely been eroded. The Appalachians are rounded hills, strikingly different from the jagged snow capped peaks for the younger mountains in the west. The deformation that produced the Appalachians gradually dies out into broad gentle folds in Pennsylvania. Erosion of these folds has left the resistant strata as ridges. From satellite or aerial views, these large-scale crustal folds are revealed by the topography and are evident. They provide illustrations for many textbooks, and are the subject of many laboratory exercises.

Undeformed strata extend westward from the Mississippi River, at approximately sea level. The strata gradually increase their elevation until they are about a mile above sea level at the Rocky Mountain front in Denver, Colorado. The mountains tower above this plain, which is already high by eastern standards. Some geological formations in the Rockies are obviously the result of one thrusting crustal slab over another. In other places, the ranges seem simply to have "popped" up. The broad Colorado Plateau west of the Rockies seems to be an area that has simply been elevated. It is cut by a few gullies, like the Grand Canyon and Canyon de Chelly, but otherwise not remarkable for its local topographical relief. Further west is the Basin and Range, which have topography that is more varied. The Basin and Range province consists of a north-trending assortment of basins bordered by stubby ranges that has been described as a herd of caterpillars heading north out of Mexico. Basaltic and rhyolitic volcanism, some of it dramatically explosive, is common and recent in this area.

Near the Pacific coast, we encounter the Sierra Nevada Mountains - an agglomeration of granite intrusions that form this impressive batholith. El Capitan at Yosemite National Park is one of the most well known of these granite intrusions. Some of the intrusions engulf large blocks of sediment, which evidently fell into the intruding mush, as the intrusions were emplaced.

Descending from the Sierra Nevada Range, we cross the San Jaoquin Valley of California, noted for its fertility and agricultural production (once irrigation is supplied!). At its western margin, 200 km north of Los Angeles, is the Carrizo Plain where the San Andreas Fault is a clear line, which offsets a small creek by 70 feet. In this location, you could literally put your finger on the boundary between the Pacific and North American plates.

The southern coast of California has the San Andreas Fault but no volcanoes. In northern California, the San Andreas Fault disappears, and is replaced by a line of magnificent volcanoes: Mt. Shasta, Mt. St. Helens, Mt. Ranier, etc. In Oregon, we cross the Snake River flood basalts. If we chose to we could follow them from near the coast across Idaho to Yellowstone National Park in the northwest corner of Wyoming.

The trip raises a host of questions: Why are there no volcanoes south of Mt. Shasta in California? Why does the San Andreas end where these volcanoes start? Why is the western part of the U.S. so much higher than the east? What formed the Basin and Range? What caused the recent explosive volcanism there? What formed the Rocky Mountains, remarkable in being in the interior of a plate, and not at a plate boundary? How and when did the Appalachian Mountains form? What sustains Yellowstone's volcanic and hydrothermal activity? You know the answers to all these questions!

When North America rifted from Europe in Triassic time (~200 million years ago), opening the rift basins in New Jersey (the Newark Basin) and Connecticut (the Hartford Basin), dinosaurs wandered about the landscape leaving their tracks. You can see these tracks if your visit that area or the universities that are there. For example, a nice set of dinosaur tracks can be seen in a large slate block attached to the wall outside the main geology lecture room in Snee Hall at Cornell University. Plumes lay along the mid-Atlantic ridge and at that time, Asthenospheric magma overflowed from these plumes and pushed North America west over the mantle. Volcanoes lined the West coast at this time, marking where the Juan de Fuca Plate was overridden and subducted beneath the North American plate. Islands and micro-continent debris on the Pacific Plate were swept (accreted) into the North American continent causing minor orogenies (Sonoma and Sevier). The Mesozoic age Sierra Nevada batholiths were produced beneath the volcanoes associated with the subduction of the "Pacific" ocean plate. The relatively low initial $^{87}Sr/^{86}Sr$ ratios (~0.707) of these batholiths suggest that relatively little crustal material or sediment contributed to their formation. The batholiths resulted mostly from differentiation of the subducted mantle lithosphere.

At the start of the Cenozoic age, in the Laramide time (~ 60 Ma ago when the Rockies were uplifted), important events happened in the western U.S. The arc volcanism swept inland, and the western parts of the continent were thrust over the eastern parts in areas like the Wind River Range in Wyoming. Intrusion-related (Porphyry) copper deposits were formed in Arizona, New Mexico and Utah.

Beginning about 30 million years ago, North America overrode two plumes: the Yellowstone plume and the Ratan plume. These plumes are now 600 to 900 km inland under Yellowstone and Ratan, New Mexico. By re-setting the age of the lithosphere to 25 Ma, these plumes caused the uplift of the western U.S. The continental divide lies everywhere within 70 km of a line joining the present positions. The divide migrated eastward with the plumes. The Snake River basalts mark the track of the Yellowstone plume. The Yellowstone plume is responsible for the hydrothermal activity and volcanism in Yellowstone National Park, and in fact is the reason this area is scenic and has been made into a park.

Both plume-induced thinning and the cessation of subduction and convergence between the 'disappeared' Farallon Plate and the North American Plate in the Western US caused the 'Altiplano' that had previously formed to collapse, allowing it to be pulled apart into the Basin and Range Province. Explosive volcanism accompanied this rupturing of the crust. The Colorado Plateau stayed simply elevated. Perhaps because it was stronger, perhaps it will pull apart into a Basin and Range topography at a later date. The Rocky Mountains were reactivated at this time, popping up along old lines of crustal weakness as the plumes and uplift swept eastward.

The termination of the San Andreas and the start of the Cascade Volcanic range is the present location of the triple junction of three plates. To the south, the Pacific and North American Plate run along the San Andreas strike-slip fault. To the north the Juan de Fuca and North American plates face each other across a trench into which the Juan de Fuca plate is being subducted. The Juan de Fuca Plate is created at a spreading center 400 km offshore called the Juan de Fuca Ridge (The Juan de Fuca Plate is the present northern remnant of the Farallon Plate, while the Cocos Plate subducting offshore Mexico and Central America is a southern remnant of this plate). A transform fault from the Juan de Fuca Ridge (the Mendocino Fracture Zone), a transform fault from the East Pacific Rise in the Gulf of California (called the San Andreas Fault), and the trench are the tectonic units that separate the plates and join to form the triple junction. The triple junction is the point at which the San Andreas ends and the volcano begin. This triple junction has been migrating northward with time. The San Andreas has been lengthening while the Cascade Chain has been shortening.

The Appalachian Mountains were formed when the predecessor of the present Atlantic Ocean closed and Europe collided with North America. The opening and closing of oceans, an inevitable characteristic of continental drift and sea floor spreading on a sphere, is called the Wilson Cycle in honor of its discoverer. Oceans close to assemble mega-continents like Pangea, only for those mega-continents to split apart again. The pieces then collide and reassemble, split apart again, etc. This basic process has probably been occurring since shortly after the Earth was formed.

The major aspects of North American geology are understood in terms of the fundamental physical processes operating in the Earth. To read the geology of North America or elsewhere, however, one has to be able to distinguish granite from basalt of sedimentary rock, determine if the sediments have been metamorphosed, and understand the implication of rock chemistry. Andesites suggest subduction, basalts rifting, and granites can have varied implications. The laboratory exercise that follows prepares you somewhat to read the rocks you find in nature and appreciate their plate tectonic implications.

Laboratory 2: Minerals and Rocks

Our objective here is to help you become familiar with common rock-forming minerals and then with representative members of the 3 rock groups that appear in the rock cycle. This necessity involves exposure to terminology specific to these fields, some discussion of the genesis of each kind of rock, and learning to distinguish characteristics that are observable in hand sample. The lab should teach you how to identify minerals/rocks, and enhance your understanding of the environments that produce different kinds of rocks.

A. Minerals

We will leave to the text the definition of a mineral and most details of chemistry and structure, and get right to the main groups. The premier rock-forming mineral group in the crust and mantle is the silicates. Non-silicate groups include carbonates (xCO_3), halides (xCl), oxides (xO), sulfides (xS), sulfates (xSO_4), and the mineral group comprised of native elements (where all crystal lattice sites are occupied by the same atom). In contrast, Earth's much denser core is a metallic Fe-Ni alloy.

We have 12 mineral specimens available, chosen because of their abundance, a certain characteristic, or their importance in a certain rock-forming environment. What you will do is handle these minerals, look at then, and apply certain tests to them to identify them (associate their name with them).

To identify a mineral, you must first list as many of its observable properties as possible, using the tools you have available. Then, use tables or flow charts to find its name. Take this general approach:

Make a table with the following column headings: Mineral Number, Luster, Streak, Hardness, Cleavage & fracture, Color, Other, and Mineral Name. (Use an 8 1/2 X 11 sheet of paper, turn it sideways so that the left (3-hole) side is at the top, and write these headings in order from left to right.)

Now, observe each hand sample and run the appropriate tests, per the instructions below (next page). See Busch and Tasa, pp. 27-32.
"Luster": Metallic or non-metallic? If in doubt, assume non-metallic.

Metallic:

Hardness:	Give it a number or a range on the Mohs Scale
Color:	Whatever it is, note it and figure it may or may not help
Cleavage	Qualities, quantities, and angles between cleavage directions
Streak:	Whatever it is, note it carefully
Special:	Note any other outstanding characteristics
Identify:	Use Table 2.27 in Busch and Tasa

Non-metallic:

Separate:	Light colors from dark colors
Hardness:	Give it a number or a range on the Mohs Scale
Cleavage:	Qualities, quantities, and angles between cleavage directions.
Streak:	Whatever it is, note it carefully
Special:	Note any other outstanding characteristics
Identify:	Use Table 2.28 in Busch and Tasa

Another couple of tests that might be of interest are reaction to HCL, (whether or not the sample effervesces ("fizzes")) and double refraction (produces a double image when laid down on a pencil line on a piece of paper).

As something of an assist, some common silicate rock-forming minerals are, in alphabetical order: amphibole, biotite, muscovite, olivine, orthoclase, plagioclase, pyroxene, and quartz. Some other common minerals are calcite, garnet, gypsum, and halite.

B. Igneous Rocks

Igneous rocks are, in principle, easy to understand and identify. In the process, we can in some cases place them in appropriate plate tectonic context.

The first step is to describe the so-called "texture" of the rock (see Busch and Tasa, pp. 55-56), estimating the percentage of "light" and "dark" minerals overall and identifying the relative abundance of two or three light ones, and using a "color Index" and the table on p. 56 or Busch and Tasa to give the rock a name. The "color" index" is attached.

Make a table on a piece of paper in the same orientation as before, but with headings Rock Number, Rock Texture, Rate of Cooling, Mode of Origin, Minerals and Mineral percentages, and Rock Name. Under Mode of Origin put "extrusive" or "intrusive," plus the appropriate letter selected from the attached Legend of Rock Origins.

Identify the rocks. Hint: how many rocks are there? How many different kinds does the table allow? Would it make sense to have more than one sample of each kind?

C. Sedimentary Rocks

Sedimentary rocks come in two flavors: clastic and chemical. By definition, sedimentary rocks are lithified sediments. Since a clast is any mechanically transported fragment of a parent material (Busch, p. 73), clastic sedimentary rocks are those comprised of lithified clastic sediments, where transported fragments have formed a layer and then undergone lithification through, in general, the application of some heat pressure. Chemical sedimentary rocks, on the other hand, are crystalline sedimentary rocks formed via the precipitation of inorganic materials from water. A third category that we will lump together with the second is that of Biochemical sedimentary rocks, where plant fragments or animal shells or skeletons provide the material of the initial layer.

General procedure:

(a) Determine whether the rock contains $CaCO_3$ (calcium carbonate)
(b) Determine whether the rock has a clastic (fragmental) or a crystalline texture
(c) Determine the grain size and the grain type (e.g. fossil or rock fragments, quartz grains, oolites)
(c) Determine the composition of the matrix

Make a table as before, with the headings Rock Number, Rock Texture, Constituents, Origin, and Rock Name. Use the attached Legend of Rock Origins to choose the best Origin description. Use the tables on pp. 77-78 of Busch and Tasa to identify the rocks.

D. Metamorphic Rocks

Metamorphic rocks are (you will recall from the quiz) either igneous, sedimentary or other metamorphic rocks that undergo changes in their mechanical character ("texture") and/or mineral assemblage via heat, pressure, and/or chemically active fluids.

Use these criteria to identify your metamorphic rock samples:

(a) Determine whether the rock is foliated or nonfoliated.
(b) If the rock is foliated, determine grain size and the type of foliation (slaty cleavage, phyllitic, schistocity, or gneissosity -- see pp. 91-93 or Busch and Tasa for explanations).
(c) Determine its mineral composition.

Due to your time constraints, (c) does not play a large part in identification here.

Make a table as before, with headings Rock Number, Texture, Origin, and Rock Name. Say what you can in a phrase about the plate tectonic environment or origin, and use the Legends of Rock, now playing in Las Vegas. Where possible, use mineral names in the rock name to make it more specific.

Legend of Rock Origins

Choose the letter that best describes the origin of the sample unknown.

Igneous:

a) erupted "non-explosively" from an ocean island volcano (e.g.: Hawaii).
b) erupted "explosively" from a volcano at a convergent plate boundary.
c) crystallized slowly from a felsic magma at depth in the Earth's crust.
d) crystallized slowly from a mafic magma at depth in the Earth's crust.
e) crystallized at depth, and was then erupted onto the Earth's surface.

Sedimentary:

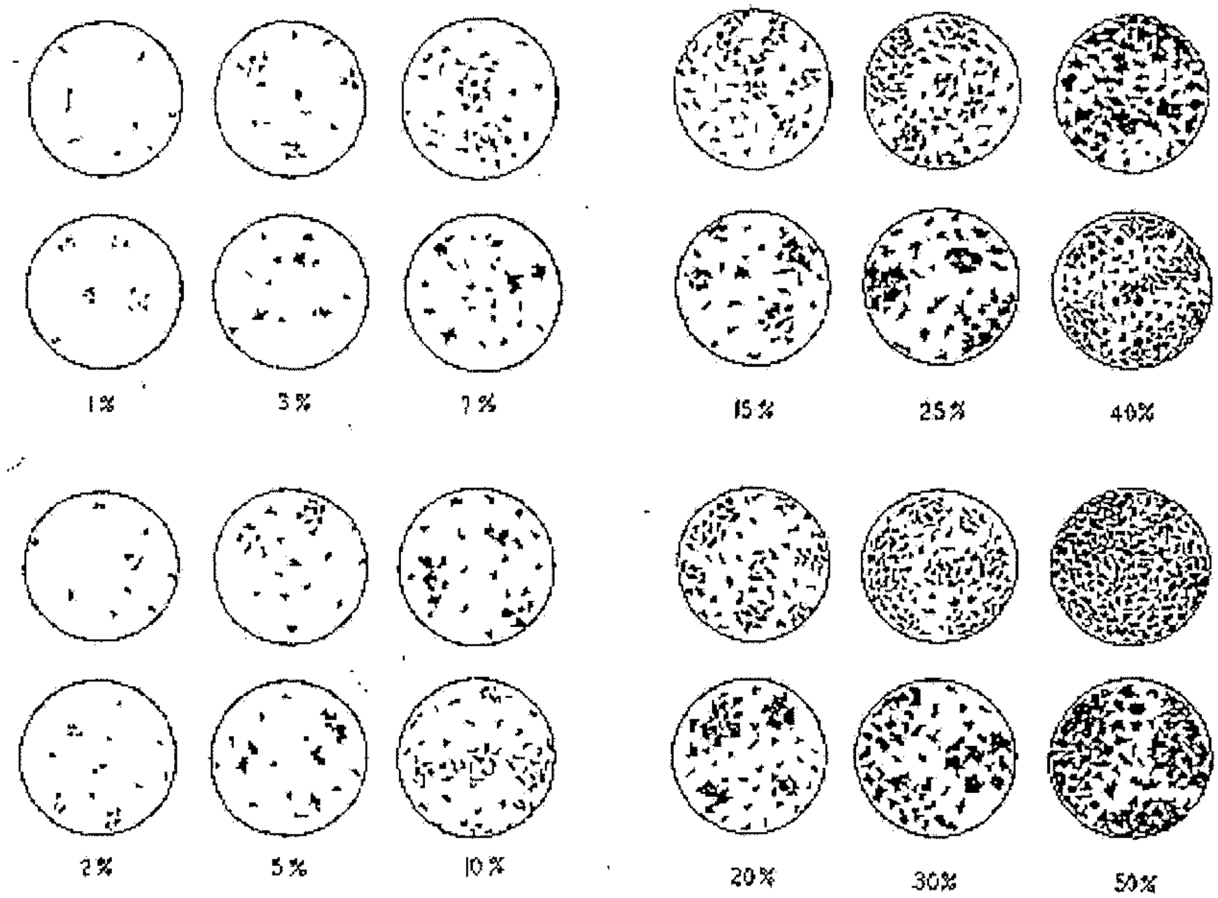
f) deposited in a moderate to high energy environment (such as on a beach) and has had a long history of transport.
g) precipitated in a shallow marine environment.
h) formed by the slow accumulation of clay-sized clasts as they settled out of a quiet body of water.
I) deposited a short distance from the source rock from which it was eroded (i.e. a short history of transport).
j) formed by the accumulation of microscopic siliceous organisms on the deep ocean floor.

Metamorphic:

k) recrystallized from a limestone protolith either in a regional or a contact metamorphic environment.
l) recrystallized from a quartz sandstone protolith either in a regional or a contact metamorphic environment.
m) recrystallized from a shale protolith under relatively low temperature and pressure regional metamorphic conditions.
n) recrystallized under conditions of relatively high temperature and pressure deep within the Earth's crust.

Use the following mineral percentage chart to help you estimate the mineralogy of your samples

COMPARISON CHART FOR ESTIMATING PERCENTILE COMPOSITION



after R.D. Terry & G.V. Chilinger,, Journal of Sedimentary Petrography, v. 25, p. 229.

LABORATORY QUIZ ON MINERALS AND ROCKS

(Tarbuck and Lutgens, Chapters 1,2,3,8)

1. The most common group of minerals is:
   a. native elements
   b. oxides
   c. sulfides
   d. halides
   e. silicates

2. What strongly influences the size of mineral crystals in igneous rocks?
   a. size of the nuclei in the crystal lattice
   b. space available for the crystal to grow
   c. amount of physical disturbance in the immediate environment
   d. heat of fusion
   e. rate of cooling

3. In order from low grade to high grade metamorphism, we see the rock types slate, phyllite, schist, and gneiss. What kind of alteration comes after gneiss?

4. Bowen's reaction series is actually comprised of two "series," the ____________________________ reaction series (on the left) from olivine to biotite mica, and the ________________________ reaction series (on the right) from calcium-rich plagioclase to sodium-rich plagioclase (feldspar).

5. Write the number of the word in the right column that is closest in meaning to the word in the left column:

________	a. extrusive	1. intrusive
________	b. lava	2. glass
________	c. plutonic	3. strata
________	d. obsidian	4. magma
________	e. beds	5. volcanic

6. Batholiths, the largest intrusive igneous bodies, are composed primarily of:
   a. granite
   b. andesite
   c. basalt
   d. rhyolite
   e. obsidian

7. The great majority (75%) of all rocks exposed at the Earth's surface are of what rock type?

8. Put an "M", an "S", or an "I" in front of each rock name to indicate whether it is a metamorphic, a sedimentary, or an igneous rock. Note that the alphabetic order in which these are placed provides no clue.

   a. gabbro
   b. gneiss
   c. granite
   d. limestone
   e. conglomerate
   f. quartz sandstone
   g. blueschist
   h. coal
   I. shale
   j. marble

9. "Metamorphism? involves the transformation of rocks by _____________, _____________. and _______________.

10. When the water in some lakes evaporates, salts are left behind. These "salt flats" are examples of _______________ deposits (be specific ... "sedimentary" is not acceptable!)

11. <u>Bonus question:</u>
    True or False: Now you are so jazzed about minerals and rocks that, if you could, you would cancel spring break in order to do our rocks and minerals lab sooner.

III. THE WATER CYCLE

So far, we have considered processes deriving energy from the Earth's interior. We now turn to processes driven by heat from the sun. The solar flux to the Earth's surface at the Earth's distance from the sun is ¼ of 1.34 kW/m^2, or 330 W/m^2, because only ¼ is intercepted by the Earth at any given time. Remember that the solar flux impinges on a disk with area πr^2 whereas the entire surface area of the Earth is $4\pi r^2$. The heat flux from the Earth's interior is 0.07 W/m^2. *Thus the energy flux from the sun is ~5500 times larger. For this reason, the sun alone* controls temperatures near and above the Earth's solid surface. The heat flux from the Earth's interior, although effective in driving plate tectonics, is negligible when it comes to heating the surface. The sun heats the surface of the Earth, drives wind and ocean currents, and evaporates water. The winds carry the water vapor over the continents where it falls as rain, recharges ground water, and ultimately runs off through rivers and streams to return to the oceans. The winds, rains, and currents erode (with some chemical help from dissolved CO_2) the mountains that internal forces have thrust up. The constant *and equal* battle between these two very different energy sources is captured in many cultures as the conflict between gods of the sun and underworld.

Chapter 9: Solar Energy, Evaporation, Oceanic and Atmospheric Circulation

The first matter of business is to understand the impact on the Earth of the solar energy flux by understanding how that energy is dissipated. How much is reflected into space, how much heats the surface, how much drives winds and ocean currents? The solar energy budget is illustrated in Figure 9.1. This figure shows that one third of the incoming solar energy is reflected from clouds, water vapor, and the Earth's surface. In other words, the albedo of Earth is 0.30. The remaining 70% of the incoming radiation is absorbed and re-radiated to space, not at the short wavelengths of the incoming radiation, but as long wavelength infrared radiation. Out of the remaining incoming radiation, 19% is absorbed by the atmosphere and 51% is absorbed by the Earth's surface. Of the 51% absorbed by the Earth's surface, 23% (i.e., 0.45 or almost half of the 51%) evaporates water (latent heat), 7% is removed by conduction or air convection (sensible heat), and 21% is re-radiated at infrared wavelengths. Of those 21%, only 6% makes it to space, and 15% is absorbed and re-radiated by the atmosphere (the greenhouse effect).

From this information, we can easily calculate the global evaporation rate. The heat required to evaporate a gram of water at 15°C is 588 cal/g. The average temperature of the Earth is 15°C and it takes a little more heat to evaporate water at this lower temperature than the 540 cal/g needed to evaporate water at 100°C. The average yearly evaporation rate over the surface of the Earth is thus:

$$\frac{\left(\pi r^2\right)\ \left(1340 W/m^2\right)\ \left(3.15\times 10^{-7}\ \text{sec/yr}\right)}{\left(4\pi r^2\right)\ \left(2360\ \text{J/kg}\right)}\times 0.23 = \ 986\frac{kg}{m^2 - yr}.$$

Since the density of water is about 1000 kg/m^3, the average evaporation rate is 0.99 m / yr over land and sea. Remarkably, the Germans estimated the average evaporation rate at ~ 1 m / yr by measuring evaporation in pans on the decks of sailing ships. The surface area of the Earth is 5.1 x 10^{14} m^2, so the total global evaporation of water is ~ 510 x 10^3 km^3/ yr.

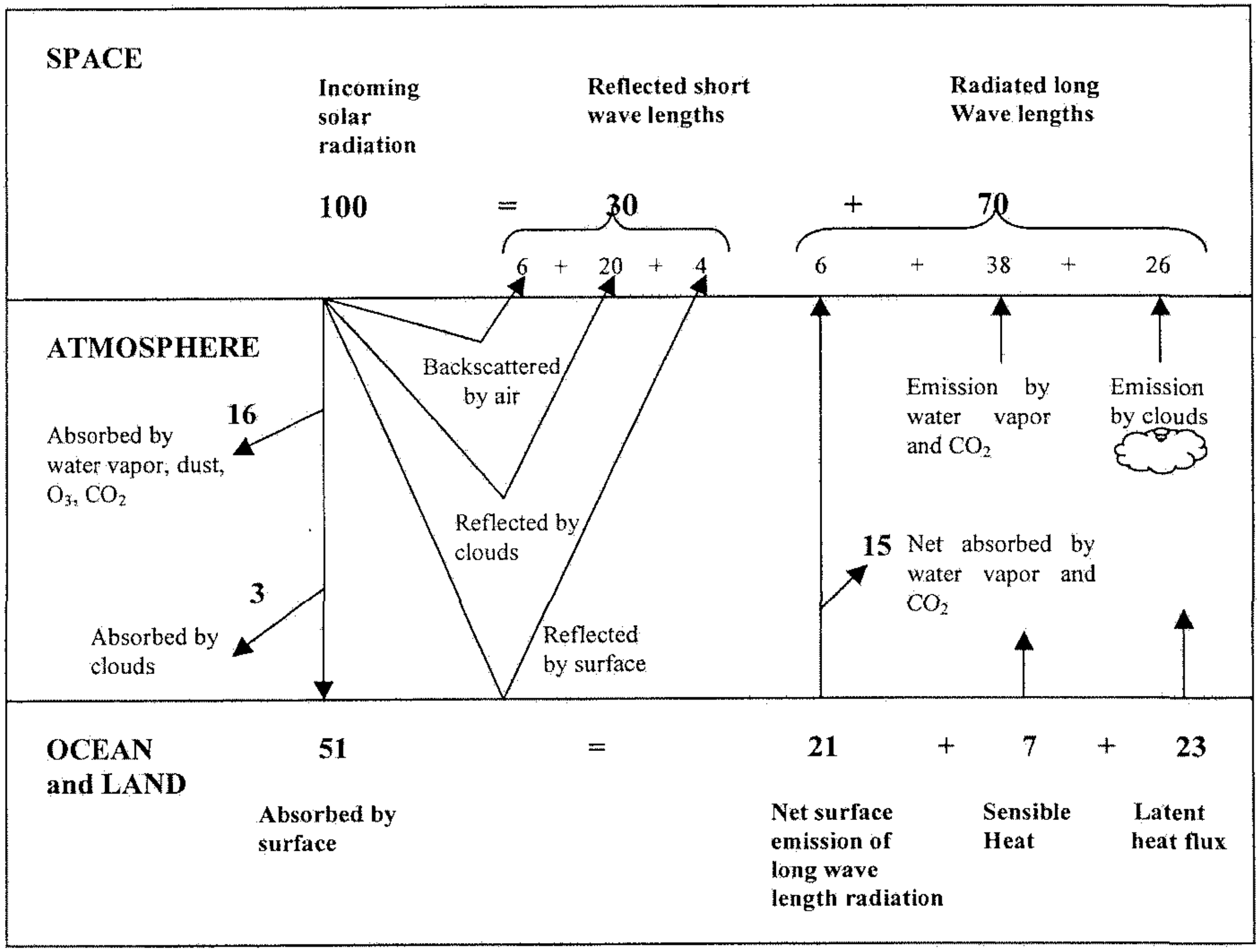


Figure 9.1. Mean annual radiation and heat balance. (Adapted from Berner and Berner, 1987)

Because the angle of incidence is oblique at the poles, the solar flux (radiation per unit area) is much greater in the equatorial regions (Fig. 9.2). The result is that the equatorial regions are hotter than the poles, but we would expect the contrast in temperature to be much greater than is observed (Figure 9.3). Winds and ocean currents act to even out the temperature of the Earth. Hadley, reasoning in this fashion in 1735, postulated that there should be a simple convective atmosphere circulation from the pole to the equator, with return flow at higher levels of the atmosphere. This is basically what happens. However, the Coriolis force prevents such a simple pattern by turning winds to the right in the Northern Hemisphere and to the left in the Southern Hemisphere. The result is that three circulation cells are required to achieve the transport of cold air to the equator (Fig. 9.4). The cells closest to the equator are called Hadley Cells and operate just as Hadley envisioned. In mid-latitudes, the Ferrel Cells circulate air north to an upwelling zone along what is called the polar front. The Polar Cells transport cold air from the South Pole to the upwelling zone on the other side of the front. Upwelling air cools and results in precipitation. Downwelling air is dry. Ithaca is in the area of upwelling along the polar front. Therefore, we have a wet climate very different from the desert climates in the zone of downwelling further south (Mediterranean and Los Angeles area).

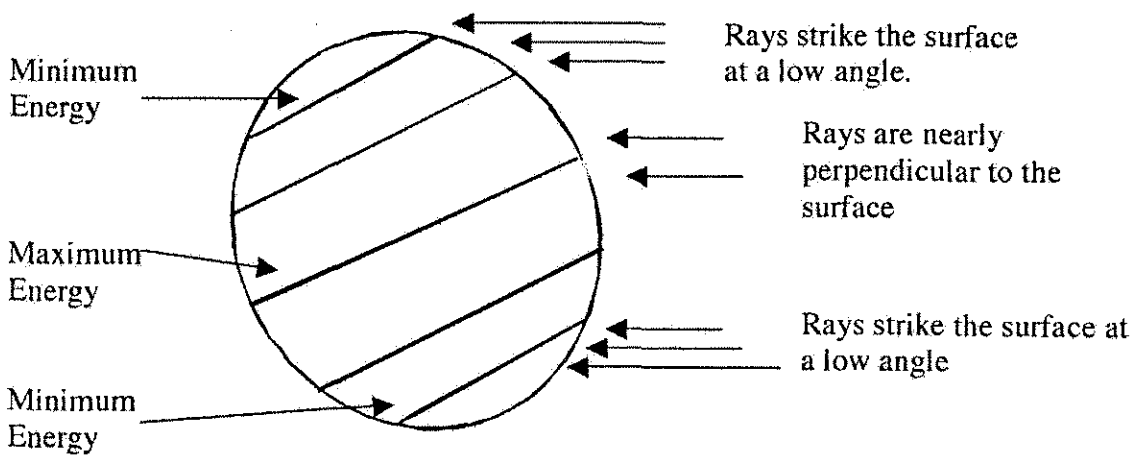


Figure 9.2 The sun's energy is unevenly distributed.
(Adapted from Earth's Dynamic Systems, p.206, 1997)

The Coriolis force turns the winds to the right or left, so proceeding north in either the northern or southern hemisphere from the equatorial upwelling (called the Intertropical convergence Zone or ITC), one first encounters the easterlies (or northeast or southeast trade winds) and then the westerlies (Fig. 9.4). These paired "trade" winds were utilized by the sailing ships of the last century to traverse the oceans in both directions. In addition, the Coriolis force is the reason for the banded appearance of the gassy giants like Jupiter. The pole-equator circulation is zonally divided there by the Coriolis force just as on Earth, only there are more zones.

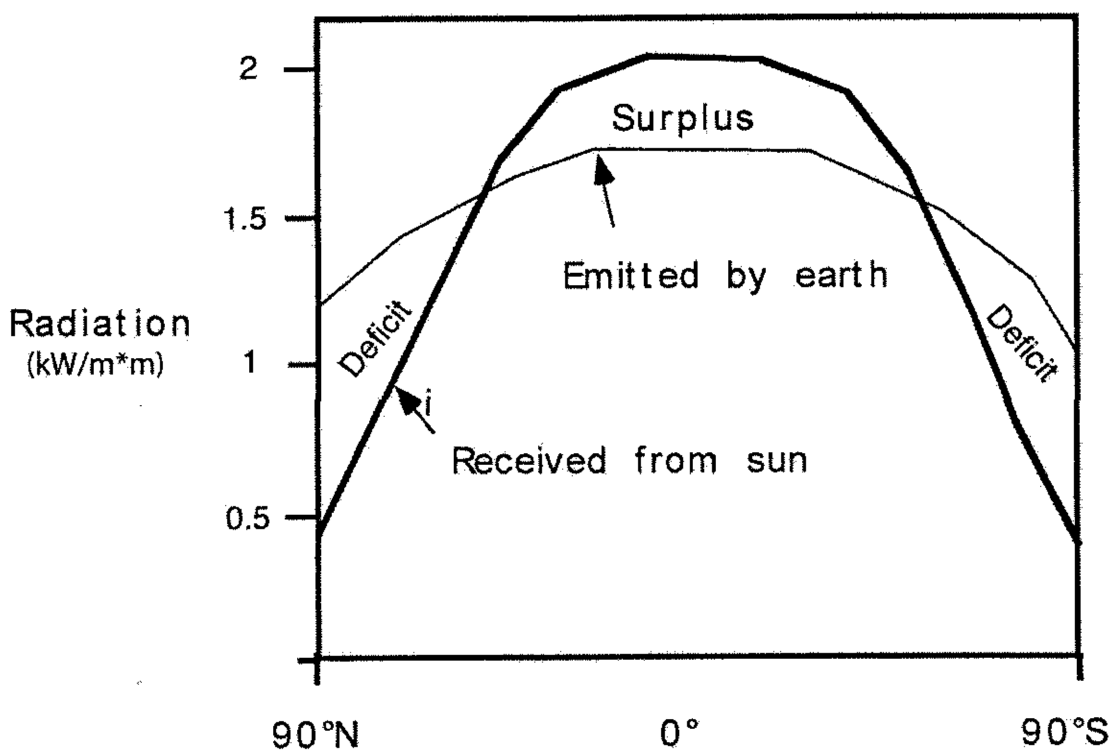


Figure 9.3. Mean annual radiation absorbed from the sun and radiated back from the Earth into space. (Adapted from Berner and Berner, 1987)

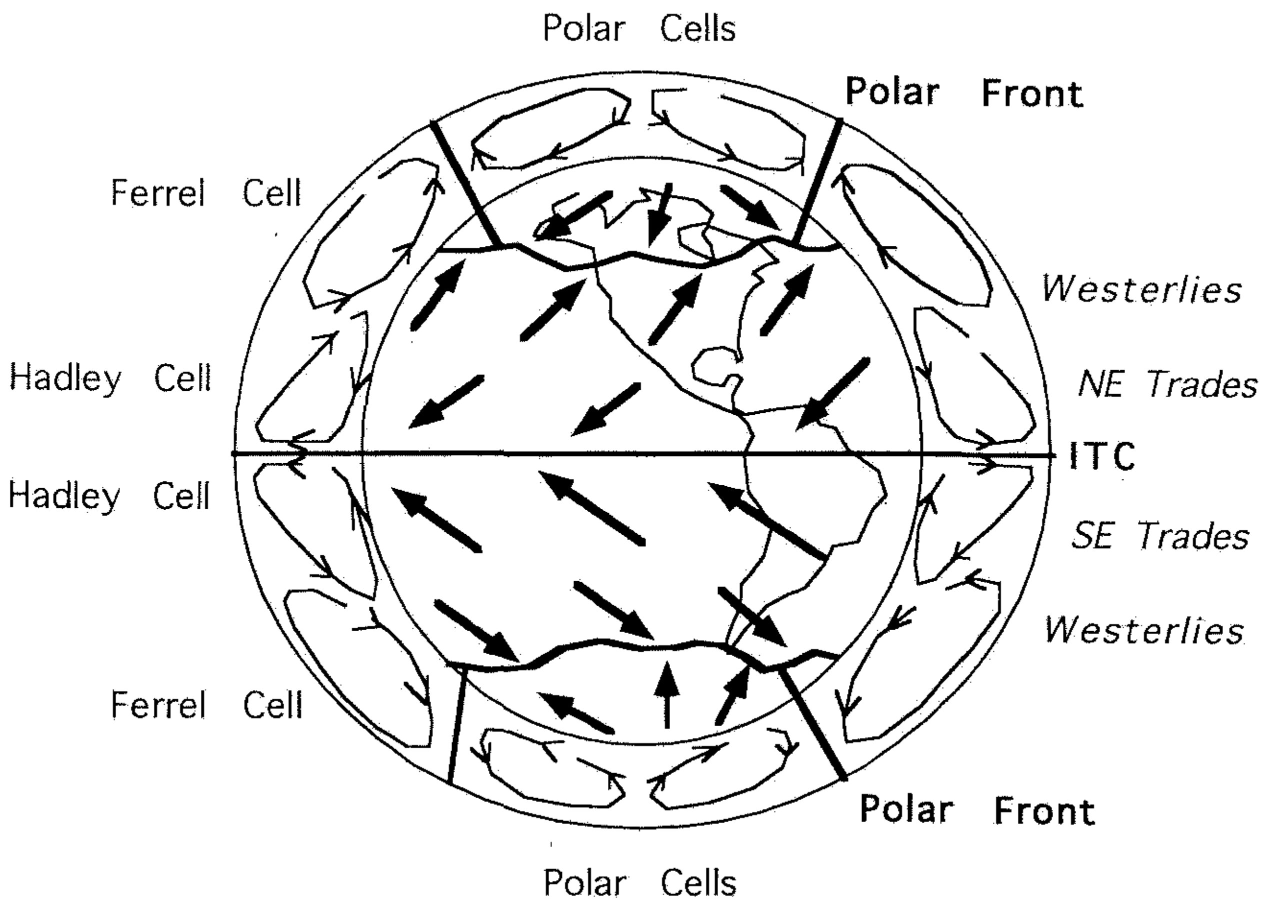


Figure 9.4. General circulation of the atmosphere. (Adapted from Berner and Berner, 1987)

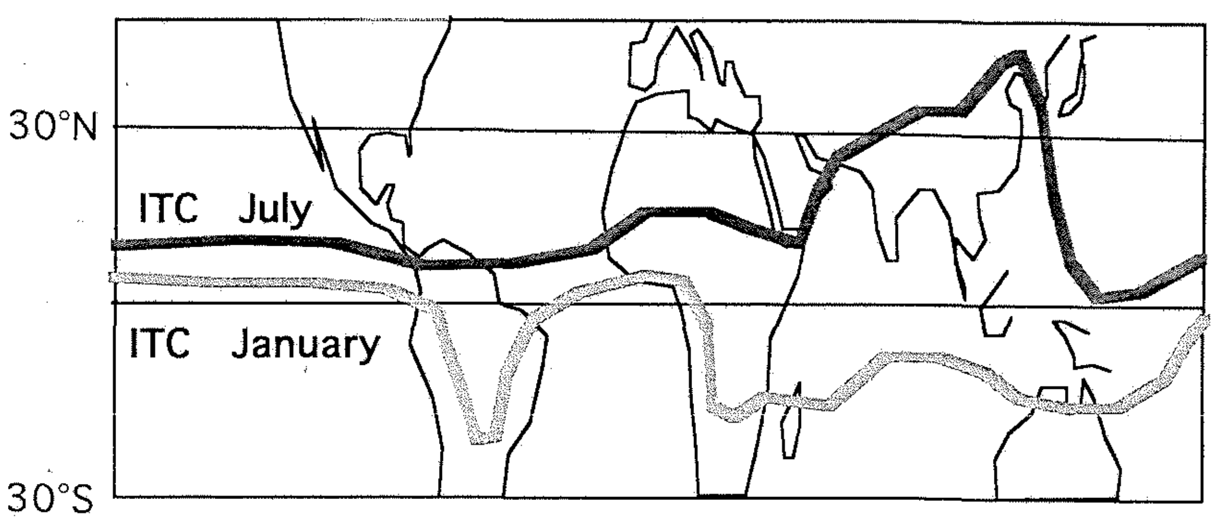


Figure 9.5. The ITC changes position with the seasons. This is responsible for the monsoons in India.

In general the ITC follows the ecliptic (the plane of the Earth's orbit around the sun) and migrates north in the summer and south in the winter (Fig. 9.5). This migration is responsible for the monsoons in India. In the winter, the ITC lies to the south in the Indian Ocean and dry air blows south from Asia, warming as it does so. There is very little rain. In the summer, the ITC lies to the north of India, and moist winds blow from the Indian Ocean over the subcontinent. The rain is practically continuous.

It has been recently recognized that the Tibetan Plateau, which you will remember was produced starting about 40 million years ago by the collision of India with Asia, has a significant effect on the migration of the ITC. In winter this extensive ~5 km high plateau is colder than the atmosphere at the same elevation, and in the summer it is considerably warmer. A strong updraft is generated over the plateau in the summer and a weaker downdraft in winter. This increases the migration of the ITC and increases the monsoonal winds. In fact, the Tibetan updraft is strong enough to influence winds and therefore ocean currents in the Pacific off the Peruvian coast. It may influence or be responsible for the El Nino current and climate variations. The solid Earth surface affects the wind pattern locally, regionally, and globally.

El Nino refers to a major shift of Pacific Ocean currents and circulation that occurs in the Pacific region every two to ten years. During a non-El Nino year, an East-West circulation pattern is present in the troposphere over this region. Consequently, easterly winds flowing over the Pacific pushes warm water westward (Figure 9.6), causing warm water to 'pile up' in the western Pacific. On the eastern coast of the Pacific, cold, nutrient rich water wells up from the deep ocean in order to replace the warm water that was transported westward. These events are considered normal and when a change occurs (possibly due to a decline in the strength of the easterly winds) the system is perturbed. This perturbation is referred to as the El Nino-Southern Oscillation (ENSO).

When the strength of the easterly winds decreases or when they change directions, there is no force to restrain the 'pile' of warm water that built up in the western Pacific. This warm water comes rushing back into the eastern Pacific, and takes 60-90 days to do so (known as the Kelvin Wave). Therefore, there is a major shift in ocean water temperature. The upwelling of cool water is no longer possible along the East Coast, which has negative consequences on the biological activity of the area. Many marine animals die or grow and reproduce more slowly due to lack of nutrients, causing the fishing industry and the local economy of the area to suffer.

The trade winds cause a global circulation of the shallow warm layers of the ocean that lie above the thermocline (Figure 9.7). (The thermocline is the region that separates warmer oxygen-rich surface water from cold oxygen-poor deep water. In the thermocline boundary layer temperature decreases rapidly with depth.) The surface circulation consists of ocean gyres that circulate clockwise in the Northern Hemisphere and counterclockwise in the southern. The western boundary currents (of the oceans) are particularly focused and intense. The Gulf Stream and the Kuroshiro Current significantly influence climate. The eastern boundary currents, where cold water flows south, tend to be more diffuse and tend to separate from the coasts, drawing nutrient-rich deep ocean waters to the surface. The "fertilized" western boundaries of the oceans are noted for their abundant marine life and fishing. The mixing time of the shallow, wind driven parts of the oceans is ~200 years.

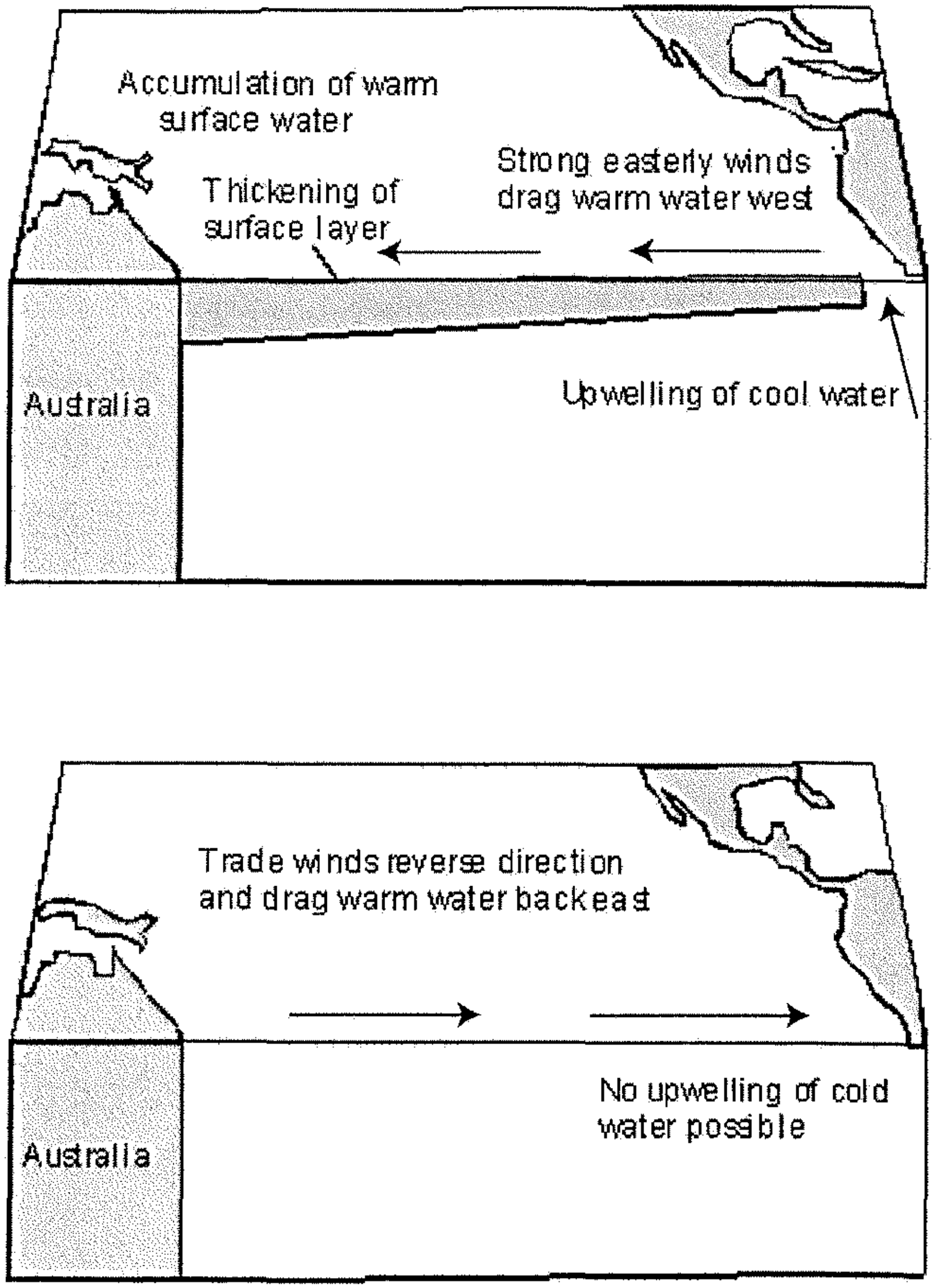


Figure 9.6 Non-El Nino year (a) and El Nino year (b) (Adapted from The Earth System, p.244, 1999)

The deep parts of the oceans also circulate on a global scale, driven by slight differences in temperature and salinity. The sources of this deep circulation are the northern and southern parts of the Atlantic. The formation of sea ice increases the salinity of the residual seawater. Cooling further increases the density of this saltier water. Cold, salty water sinks in both the Arctic and Antarctic and circulates toward the latitude of Argentina. There it turns east to the Indian Ocean and ultimately flows into the Pacific (Fig. 9.8). The deeper parts of the ocean are layered depending on density (and water source) (Fig. 9.9). The deepest waters are the Antarctic Bottom Waters. The next layer is the North Atlantic Deep Water, followed by water from more northerly parts of the Antarctic, called the Antarctic Intermediate Waters. Finally, at about the same depths, are the shallow, salty, warm waters from the Mediterranean. The turn over time of the deep ocean waters is about 1000 years.

Together winds and ocean currents transport about 270 W/m^2, or 20% of the total incoming solar energy from the equator to the poles (Fig. 9.10). Ocean currents (shallow and deep) transport 20 to 25% of the heat, winds transport the remaining 75-80% as warm air and water vapor (latent heat) (Fig. 9.8).

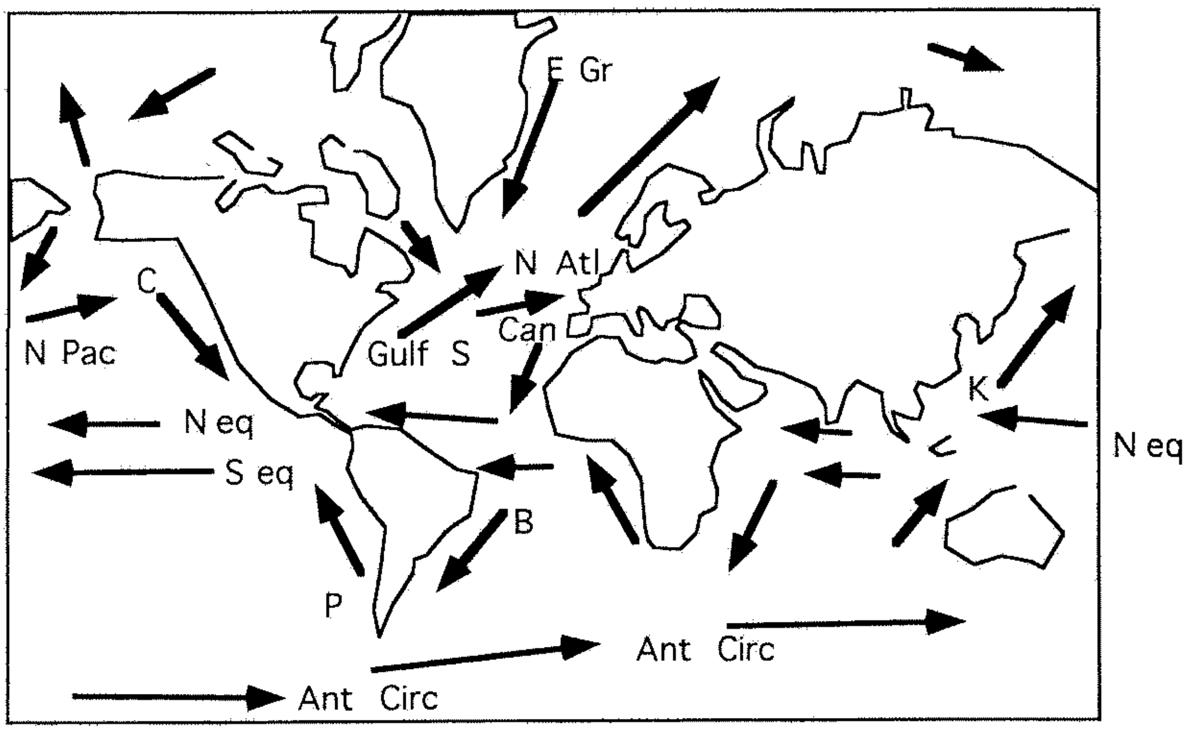


Figure 9.7. Surface currents of the oceans.

Gulf S = Gulf Stream
C = California Current
E Gr = East Greenland Current
N Pac = North Pacific Current
N eq = North Equatorial Current
Ant Circ = Antarctic Circumpolar Current
K = Kuroshiro Current
Can = Canary Current
N Atl = North Atlantic Current
S eq = Southern Equatorial Current
B = Brazil Current
P = Peru Current

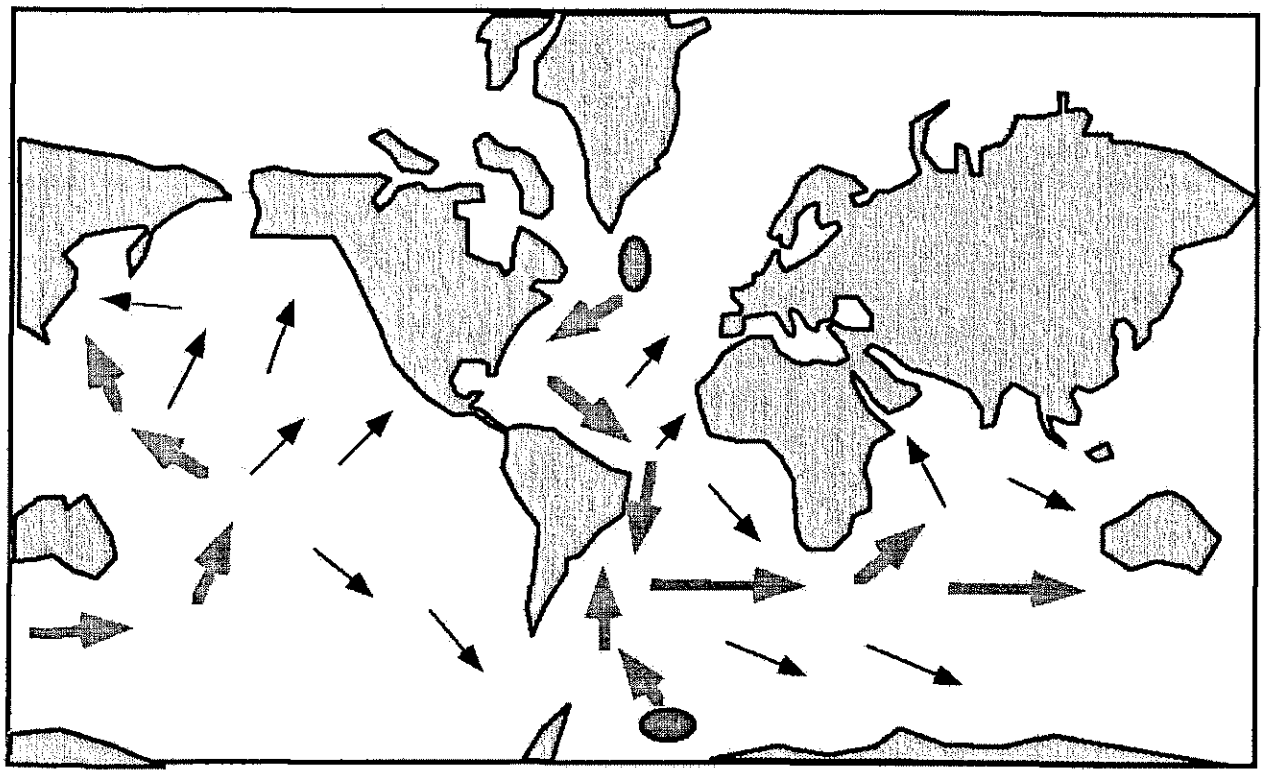


Figure 9.8. Deep (thermohaline) circulation of the oceans.

Sources are sinking saline water in the arctic and Antarctic (large stippled ovals). Flow is intense along western side of ocean basins. Note global character of circulation. (Adapted from Stommel, "Circulation of the Abyss, Scientific American, 1958)

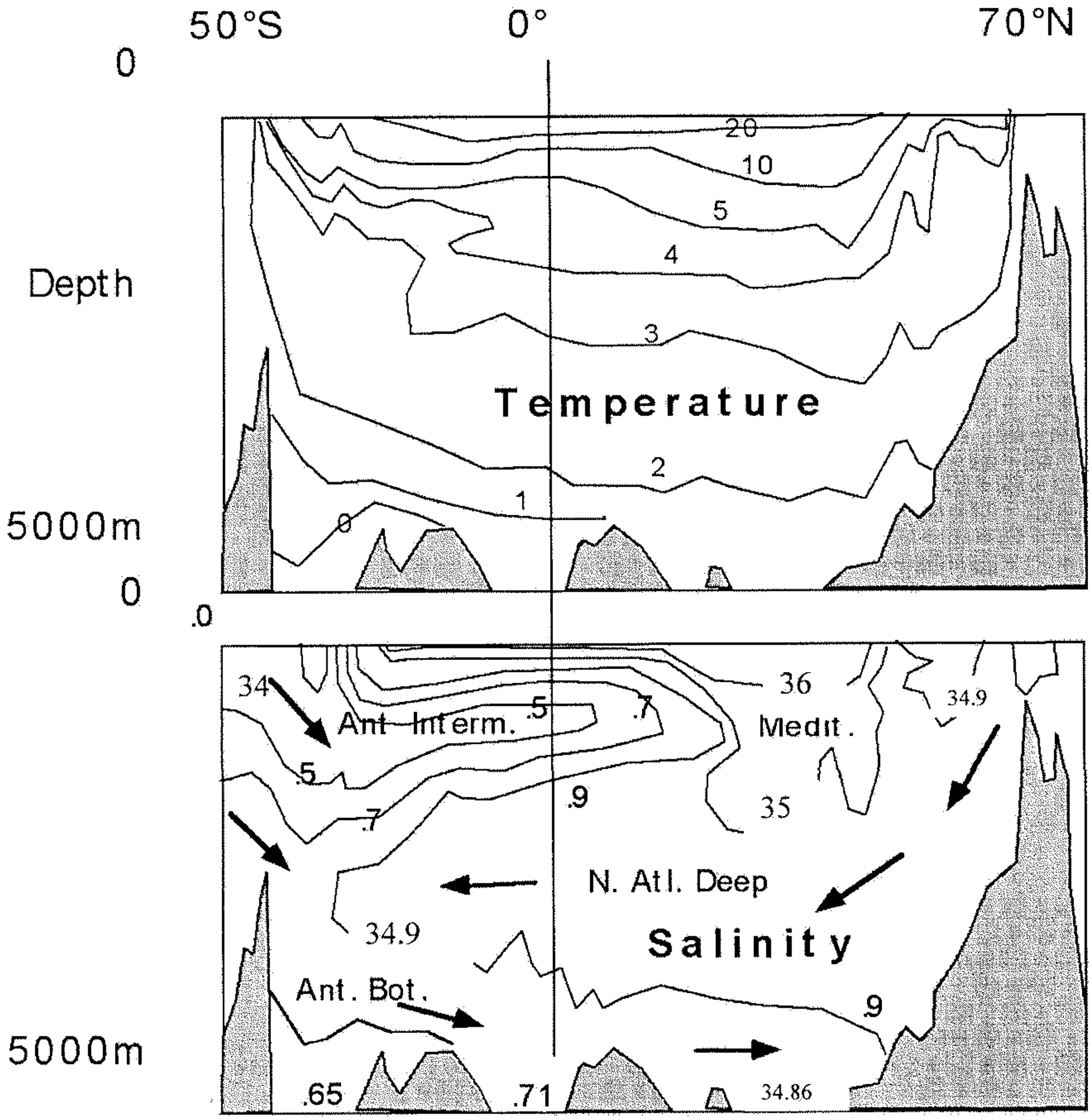


Figure 9.9. N-S vertical sections showing temperature and salinity of the Atlantic Ocean.

Water types are delineated by salinity in parts per thousand total dissolved solids. Note some of the labels give only the decimal fractions. Thus .65 means 34.65 ppt tds. (Adapted from Berner and Berner ,1987).

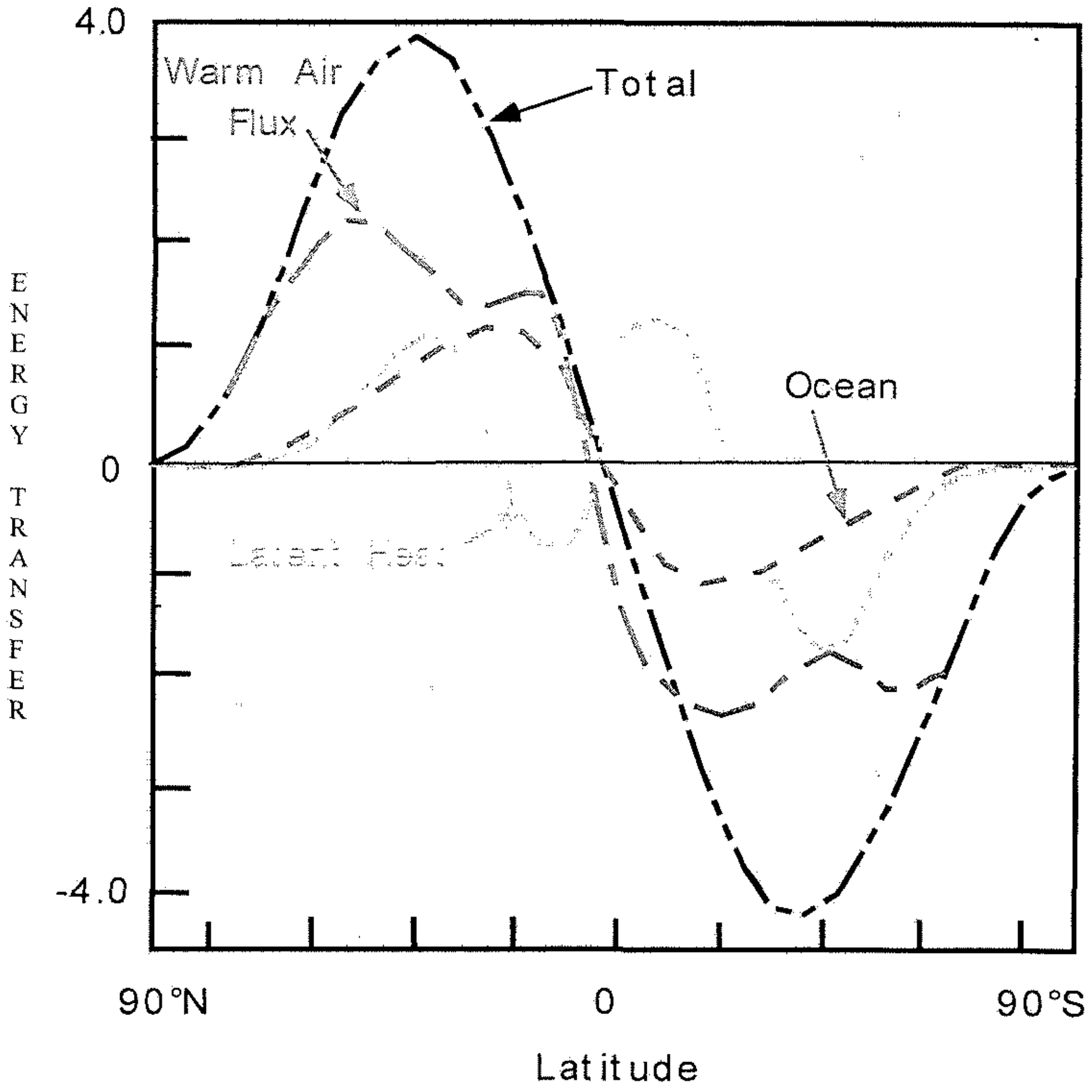


Figure 9.10. Energy transport. Positive is northward, negative southward transport. (Adapted from Berner and Berner ,1987).

Problem Set 9: Climate Energetics

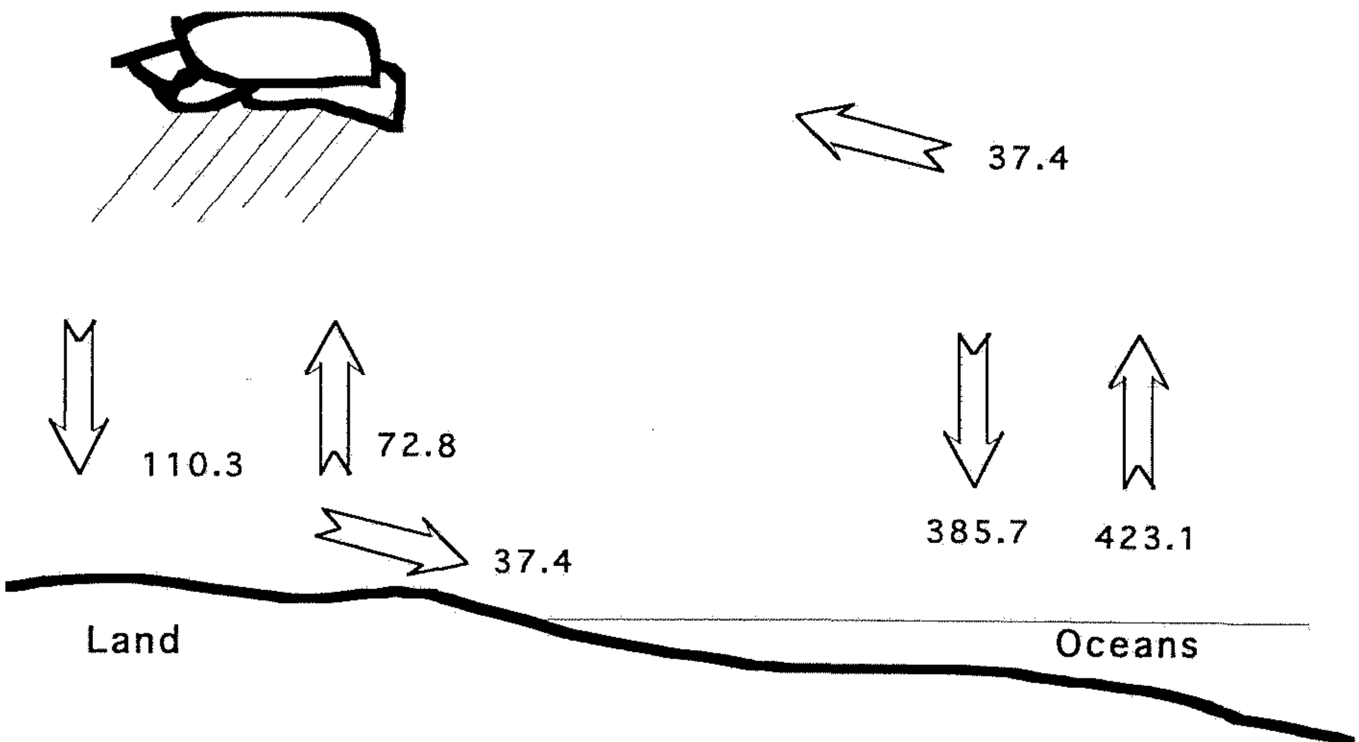
9.1 What annual fraction of heat at 70°N comes from sunlight?

9.2 What are the contours in Figure 9.9?

9.3 In what sense must the interactions between energy from the interior and energy from the sun "exactly balance"?

Chapter 10: The Hydrologic Cycle and Human Water Usage

Because oceans cover 71% of the Earth's surface, most of the evaporation and rainfall occurs over the oceans. In fact, only about 37.4 x 10^3 km^3 of the 423 x 10^3 km^3 of water that evaporates from the oceans each year is transported by winds to precipitate over the continents. This runs off to the oceans almost entirely through the world's rivers.



Fluxes in the Hydrologic cycle (in 10^3 km^3 / yr)

Figure 10.1

Over the continents there is a very important ⅓, ⅔ rule for evaporation and rainfall (Figure 10.1). One third of the rain that falls runs off, ⅔ evaporates. One third of the rain that falls comes from the oceans, ⅔ is derived locally from continental evaporation. Three times the net flux of water from the oceans precipitates each year over the continents. Two thirds of this evaporates again, ⅓ runs off to the oceans. The continental precipitation is 110.3 x 10^3 km^3 per year and runoff 34.7 x 10^3 km^3/yr.

The ⅓, ⅔ rule has some significant human implications. Most of the evaporation that takes place over land is facilitated by plants, through a process called transpiration. Plants literally suck water out of the subsurface. Utilizing capillary forces they can produce pressure in their roots equivalent to ~15 atmospheres before they wilt. The water they draw out of the ground, like the water we drink, is exhaled to the atmosphere through their leaves. Evaporation over a forest is as rapid as over an ocean or lake; in both cases the limiting factor is how fast winds can remove the water-saturated air or the boundary layer which is the immediate layer above the ocean/lake or forest canopy. (also termed boundary layer transport)

What this means is that man can drastically affect climate if he significantly alters the vegetative cover. An area of moderate rainfall can be turned into a near desert, and it will be difficult to rapidly (e.g., on a human time scale) recover the original condition short of deliberate and expensive irrigation. The magnificent spruce stands in Lebanon were cut down for shipmasts in the 1700 and 1800's. This affected the rainfall budget sufficiently, and the stands have not and will not grow back. Rainfall in the Amazon is largely of local, plant transpiration, derivation. The slash and burn practices in the Amazon rain forests that are being employed to clear the trees and produce pastures can and will significantly affect the rainfall and climate in the Amazon.

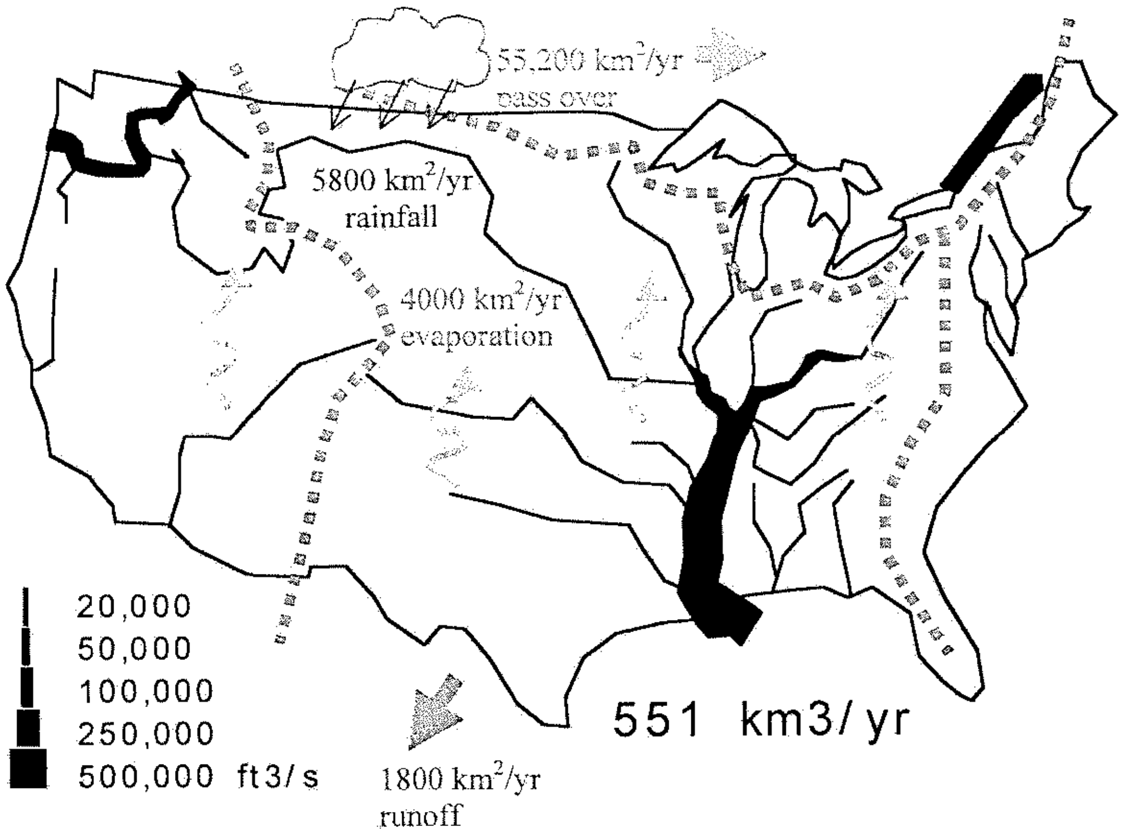


Figure 10.2. River discharges in the lower 48 States. Dashed boundaries are drainage divides. (Adapted from U.S.G.S. Water Supply Paper #2250, 1984)

Consider now the water budget in the U.S (Figure 10.2). Each year 55.2 x 10^3 km^3 of water vapor pass over the U.S. Rainfall totals 5800 km^3/year, evaporation is 4000 km^3/yr and runoff 1800 km^3/yr. Human usage is 524 km^3/yr and is supplied by groundwater. The rate of human usage is about equal to the flow rate of the Mississippi River (551 km^3/yr). Human use of groundwater for agricultural, industrial, and household purposes leads to an evaporation rate of 140 km^3/yr; the balance of 384 km^3/yr is returned to groundwaters or streams. This discharged waste water is of course available for re-use if it is still pure enough.

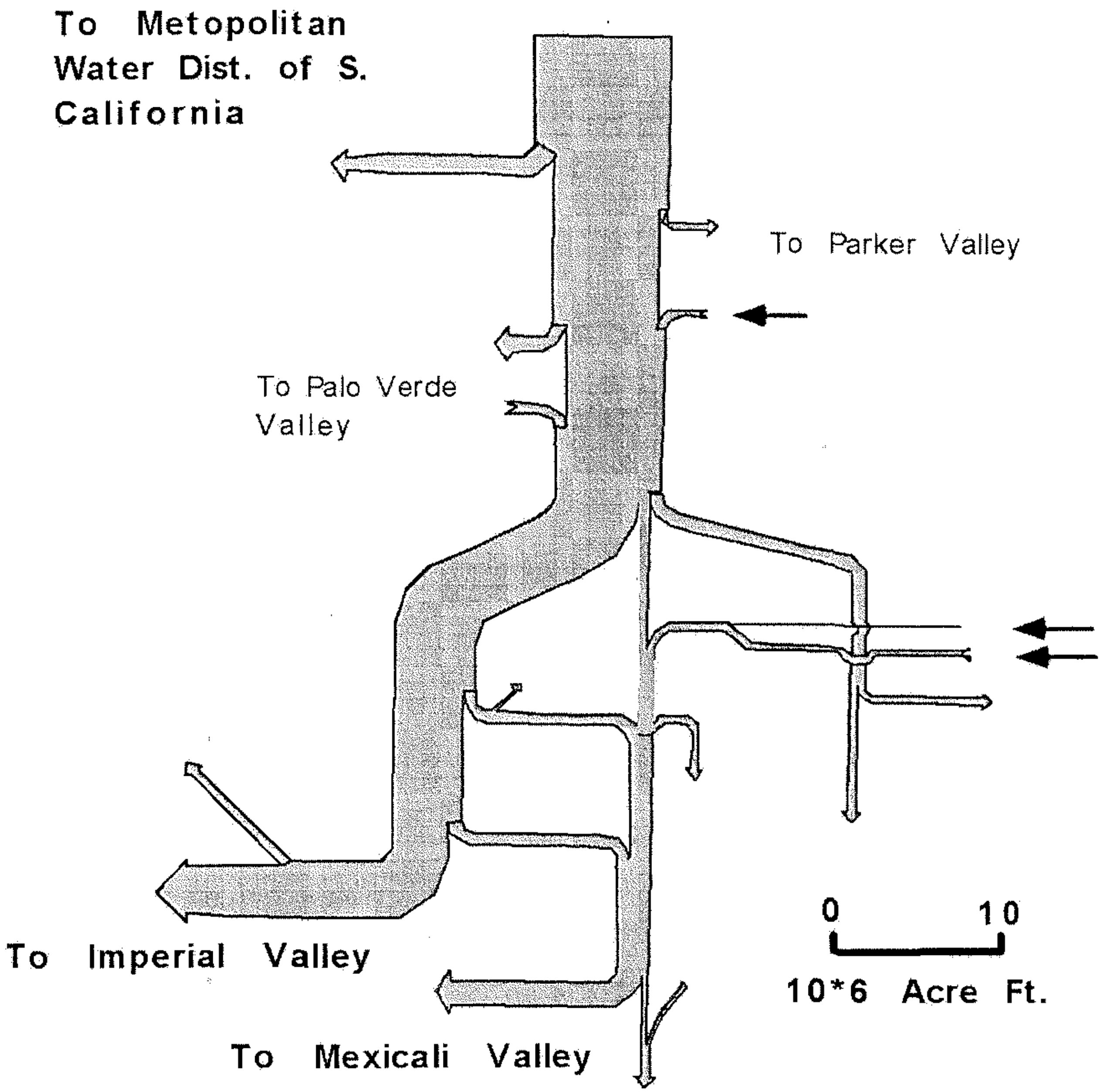


Figure 10.3. The fate of the Colorado River illustrates the water problem in arid areas. (Adapted from U.S.G.S. Water Supply Paper, #2250, 1984)

The 140 $km^{3/}$yr that humans actually use each year is only 8% of the renewable water supply of 1800 km^3/yr. In addition, by cloud seeding or other means, a larger percentage of the 55,200 km^3 of water vapor that crosses over the U.S. each year might be encouraged to precipitate. The percentage is presently only ~10%. Overall there is thus no lack of water. The problems are that : 1) the water is not evenly distributed, and 2) the water that humans return (at nearly the flow rate of the Mississippi River), after use, is often contaminated.

The first problem is illustrated by the fate of the Colorado River (Figure 10.3), which used to flow to the sea. Today it is entirely intercepted by irrigation canals. The river water is spread on fields and evaporated. The relatively small percentage that makes it to Mexico does so only because it is protected by treaty agreements. Wars are fought over riparian rights (the right to the delivery of water and the associated right to limit upstream development). Riparian law is an important branch of law, and one that will become increasingly important as the world's population grows in arid areas (Fig. 10.4). Fig. 10.4 shows that water use in the U.S. is growing somewhat faster than the population.

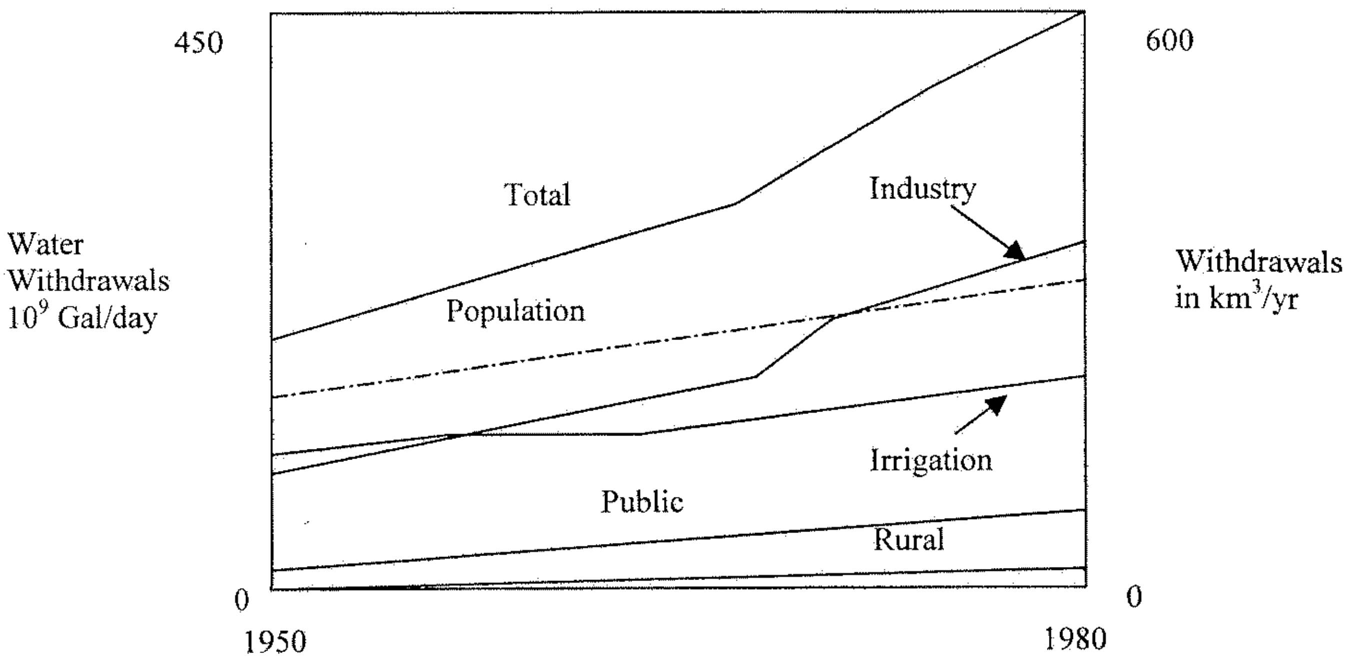


Figure 10.4 Water usage in the U.S. (Adapted from U.S.G.S Water Supply Paper #2250, 1984)

The second problem, groundwater contamination, is one that we shall consider in some detail starting in Chapter 11. For now it will suffice to point out that the residence times (average length of time a substance spends in a reservoir at steady state) of water in the subsurface is relatively long. Almost all the water that falls as rain enters the subsurface. Very little falls directly into stream channels or runs off over the surface to streams without soaking into the ground. The amount of water in the subsurface is conservatively estimated at $8{,}400 \times 10^3$ km^3. By contrast the amount of water in lakes and rivers is only 200×10^3 km^3, and the water vapor in the atmosphere at any time is only 13×10^3 km^3. Assuming all runoff moves through the subsurface, the residence time of water in subsurface aquifers, lakes and rivers, and the atmosphere is:

$$\tau = \frac{\text{reservoir size at steady state}}{\text{inflow or outflow rate}}$$

$$\tau_{aquifers} = \frac{8{,}400 \times 10^3\ \mathrm{km}^3}{37.4 \times 10^3\ \mathrm{km}^3/\mathrm{yr}} = 224.6 \text{ years}$$

$$\tau_{lakes\ and\ rivers} = \frac{200 \times 10^3\ \mathrm{km}^3}{37.4 \times 10^3\ \mathrm{km}^3/\mathrm{yr}} = 5.3 \text{ years}$$

$$\tau_{atmosphere} = \frac{13 \times 10^3\ \mathrm{km}^3}{396 \times 10^3\ \mathrm{km}^3/\mathrm{yr}} = 12 \text{ days}$$

These residence (or replacement) times tell us a good deal about the dynamics of these systems. They tell us, for example, that weather is likely to change on a daily or weekly time scale, that large lakes can change their water levels on a decade time scale, but that contamination of aquifers may be delayed for centuries! The long delay between the actions that contaminate groundwater and the manifestation of the consequences of these actions make it particularly difficult to deal with groundwater problems.

References:

1. Berner, E.K. and R.A. Berner, 1987, The Global Water Cycle, Prentice Hall, New Jersey, p. 12-47.

2. USGS Water Supply Paper #2250, 1984.

Laboratory 3: Simulating Subsurface Fluid Flow

Objective: To understand the relationship between surface topography and subsurface ground water flow by running computer simulations.

Background: We have seen how flow in the mantle is produced when the top surface of the mantle is deformed from its flat, constant elevation, isostatic equilibrium condition. Ground water flow is (for the most part) caused by exactly this same kind of boundary perturbation. Subsurface water flow occurs when the water table (the interface between water-saturated sediments and air-filled sediments) is not flat.

The water table tends to lie a roughly constant depth below the surface, and therefore to reflect topography. You can easily imagine why this might be the case. The ground water table is "pegged" at streams, but rain falls in the hills separating river valleys, and some of this rain infiltrates or sinks into the ground and drains down to the water table. This addition of water raises the water table, and causes flow that transports that water, eventually, to a stream, which delivers it to the ocean, completing the hydrologic cycle.

Ground water flow driven by elevation differences in the water table can be simulated by solving Darcy's law on a computer. Just as with mantle flow, it is convenient to start with a simple sinusoidal deformation of the water table. After doing this, complications can be added. We can solve for flow with more complicated water table shapes, insert more permeable faults and strata in the subsurface, etc. The program you will use in this lab allows you to do all of this.

The lab consists of simulating the cases described below and answering the questions in this handout. From doing this you should acquire an appreciation of the style of fluid movement in the subsurface. We will address this flow further in lecture and in the lecture notes.

Operation of the Computer Program:

To run the lab computer program you will need to know how to open and quit from the program. A few notes on its operation are also useful.

1. **Login**
   Geo201
   Water
2. **Start and exit**
   Start: double click on Flownted.exe icon
   Exit: escape q
3. **Typical commands**
   Page up, down, home to page though data entry pages
   Up arrow, down arrow, right arrow, left arrow to move though data pages

4. **Special notes**
   F1 key must be pressed before and after entering equations describing Groundwater table.
   You must exit the material property menu with a down arrow.
   Material property symbols "," and ";" cause plotting to be suspended (simulate valleys).
   Flownet can be animated if time lines are turned on.

Lab Exercises:

A. Default Case

First simply run the default case. Start up the program and page though all the text and data entry pages using the page down key on your computer until you arrive at the final plot. You should see an image of flow (streamlines and iso-head contours) of a 200 m long by 100 m deep portion of the subsurface. Flow is driven by a 1 meter co-sinusoidal elevation of the water table (shown by boxes above the rectangular computation domain).

The streamlines are solid lines.

The iso-head contours are dotted lines.

Water in a well drilled to one of the iso-head contours would stand at the elevation where that iso-head contour exits the surface (height of the box at the top). You can see how fluid pressure decreases from left to right across the section.

Ground water circulates parallel to the solid streamlines.

Flow can be visualized by turning on the time markers. To do this page back to the last data spec page before the diagram by hitting the page up button once. Use the down arrow to move your cursor to the 12th line. Use the right arrow to turn on the time step option. The line should now read "time steps drawn", rather than "time steps not drawn". Notice that the time step interval is 1 year.

Now page down to see the plot again. It should contain dots along the streamlines indicating the age of the water at each location. The age of water along a streamline can be determined by counting the dots from the recharge end. If there are 12 dots the water took 12 years to reach that point and is 12 years old.

If you hit the "+" key you can animate the dots so that they appear to move like the water is moving. The rate of movement can be slowed and even stopped by hitting the "-" key.

Finally, it is convenient to connect the dots of similar age to form isochrons. This can be done by paging back and turning the isochrons by positioning the curser on the 16th line and using the right arrow to turn the isochrons on. Now hit the page down and the "+" to see the isochron animation. You can now see how water of the same age enters and moves through the subsurface.

What is the age of the water exiting from the third streamline from the bottom? You will need to turn the isochrons off and the timesteps on, and count the dots along this streamline.

The rate of flow for the same topographic drive and subsurface permeability depends on the volume fraction of the sediment that is water-filled pores (the sediment porosity). This is a parameter that can be changed. Page up 2 pages to the hydraulic conductivity parameter page, and notice that porosity is specified on the 6th line. Actually the hydraulic conductivities and porosities are specified at a grid of points indicated by the box of "x". To change the porosity of the whole grid, hit return. This brings up the specifications of the symbol "x". Use the down arrows to get to porosity and change it to 0.15 rather than 0.3. Page down to the parameters specification page and set the time steps on (12th line). Now step down to the solution page.

Count the time steps and determine the age of water exiting from the third streamline from the bottom. You should get an age approximately half the age you previously determined. Why?

Now page back up 2 pages to the Hydraulic Conductivity page, and reset the permeability of some of the cells. For example setting the first 13 cells in the bottom 2 layers to "y" by moving to the bottom left cell and typing "y". This will cause the permeability screen to appear. Set the horizontal and vertical permeability of the "y" cells to 10 (it is presently 1 m/day). Hitting return brings you back to the grid specification. Move to the bottom left cell and hit "y" and then using the right and left shift arrows to move around the grid, "painting in" "y" symbols in the first 13 cells of the bottom two rows. When you have finished, hit return, and page down, etc., to view your new solution.

Turn on the time steps and determine the age of the water exiting from the third streamline from the bottom. It should be younger than water exiting the third streamline from the bottom. Why? What is the age of the water exiting the 3d streamline from the bottom?

Finally, it is possible to change shape of the ground water table. This can be done in two ways: with a formula and by specifying the elevation of the water table of each cell by hand. To see this page up three pages to the Hydraulic Head specification page. Notice that the head has a cosinusoidal form and plots half a wavelength. Hitting "F1" allows you to view (and change the formula). Hit "F1" and notice how the head is specified as a function of node number over the 16 nodes of the problem.

Let's specify the head by hand, however, not by formula. Hit "F1" top exit from the formula menu. Use the down arrow to move from cell to cell and change the head specified in the first 5 cells to 1, and the head in the remainder to 0. Page down and look at your solution.

Now specify the head distribution using a formula. Page back up to the Head Form, and hit "F1". Type in a formula that specifies 1.5 rather than 0.5 wavelengths of head variation by changing the denominator of the base case expression from 15 to 5. Hit "F1" (to save the new formula and display the new head variation) and then page down to see the solution.

Areas with elevated head represent the elevation of ground water table under hills. Each hill and valley establishes its own circulation system. Notice that there is a limit to the depth of flow, just as in the case of a sinusoidal perturbation of the mantle surface.

To roughly what depth does ground water flow persist? What fraction is this of one wavelength of head variation?

B. Depth of Ground Water Flow

The base case is interesting but its dimensions are not very realistic. Go the first menu, the Parameter menu, and change the number of columns to 10, the number of rows to 10 (maximum resolution) and the length of the system to 2000 m and the depth to 1000 m.

On the second menu change the head distribution to a cosinusoidal variation superimposed on a ramp so your formula is:

x*0.1 + cos pi(x-1)/5.

Hit "F1" and page down to the solution. You will see that each valley now has its on circulation system, but there is a regional flow from right to left across the diagram produced by the regional slope (e.g., the 0.1*x term in your equation). Turn on the time steps (but set the time increment to 100 years). Animating the time steps shows how the slow regional flow works.

Explain briefly how the formula works. What does x represent, for example?

Count the number of time steps in the regional (lower) streamline in the solution above. It should take thousands of years for the flow to travel the 2 km width of this section. Contaminant traveling with this flow would travel even more slowly. It could take thousands of years for Dryden's contamination to reach Ithaca. Dryden is topographically higher than Ithaca so some flow could move from Dryden to Ithaca, but contamination spilled in Ithaca would be very unlikely to flow to Dryden.

Why?

Experiment now by adding permeable and impermeable subsurface layers in this complicated flow situation with several hills and valleys. Write brief notes of what you find below. You should experiment enough to appreciate that subsurface flow is strongly controlled by variations in the elevation of the water table and by permeable and impermeable layers and (vertical) barriers or conduits in the subsurface. You may gain some inspiration from the attached figures.

What could produce vertical barriers and conduits?

Faults in fact can be both permeable (parallel to the fault) and impermeable across the fault. You can simulate this by specifying different horizontal and vertical permeabilities.

Chapter 11: Subsurface Fluid Flow

Terminology related to precipitation and subsurface fluid flow is illustrated in the following diagram (Figure 11.1). Water is stored in the subsurface in areas where the water is "perched" above impermeable layers (labeled "interflow" in the diagram), and below the water table. The water table is a region of water-saturated soil. The seepage of ground water to streams is what keeps them flowing between storms. In many areas, ground water is an important source of potable water. Where this is the case, understanding its subsurface movements is important. This understanding was first achieved by Henry Darcy. In this chapter, we seek to understand the foundations and application of Darcy's Law.

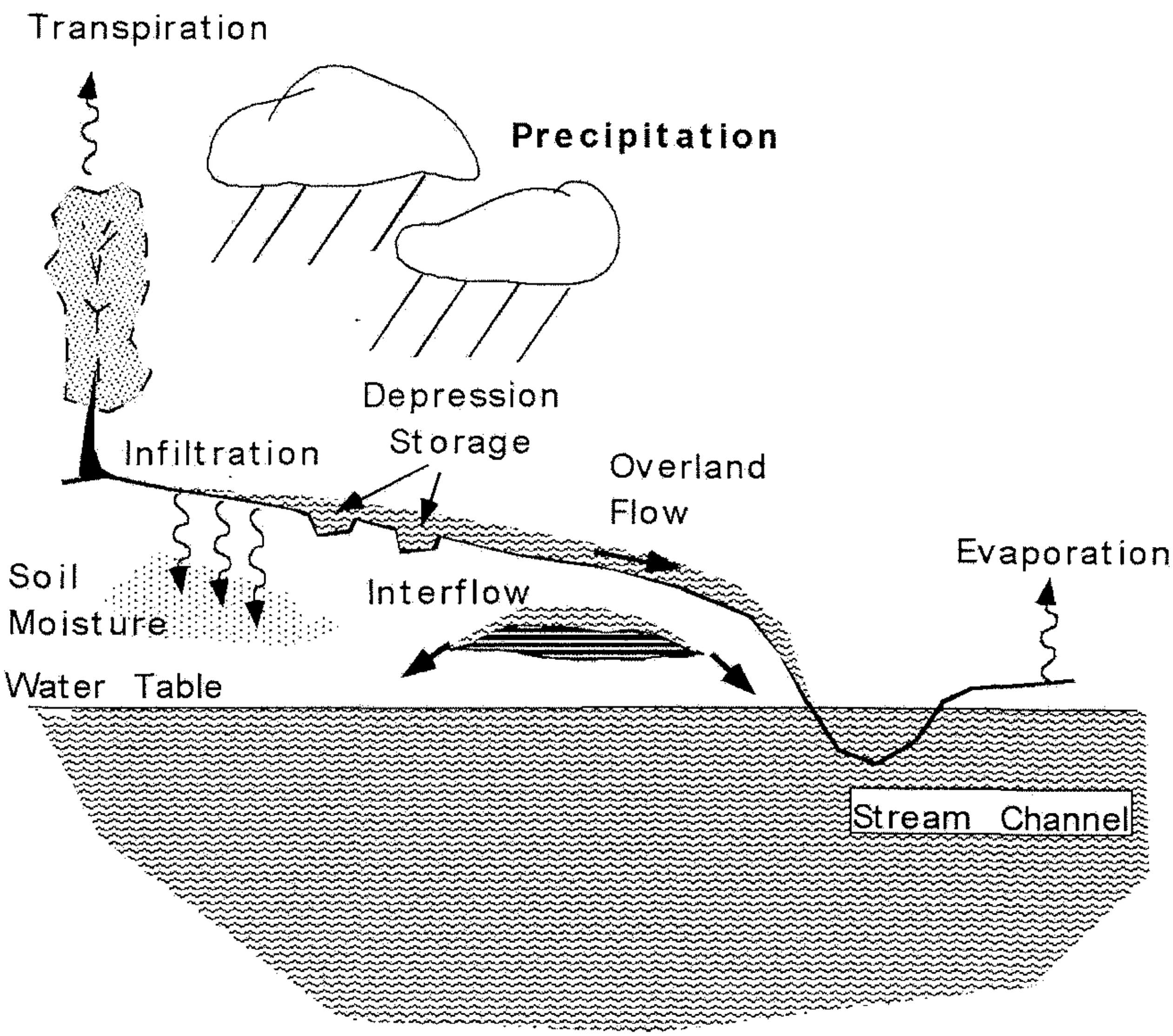


Figure 11.1 Subsurface Fluid Flow
(Adapted from Davis and DeWiest, 1966)

The first wells were undoubtedly drilled in Asia, a continent with many dry regions and a large population needing water. The ancient Chinese are known to have developed a wooden churn drill similar in design to modern drills. It took years, even decades, but wells drilled by this technique reached 1200 to 1500 m depth. Not until the end of the 19th century did Europeans equal the Chinese in this important technology.

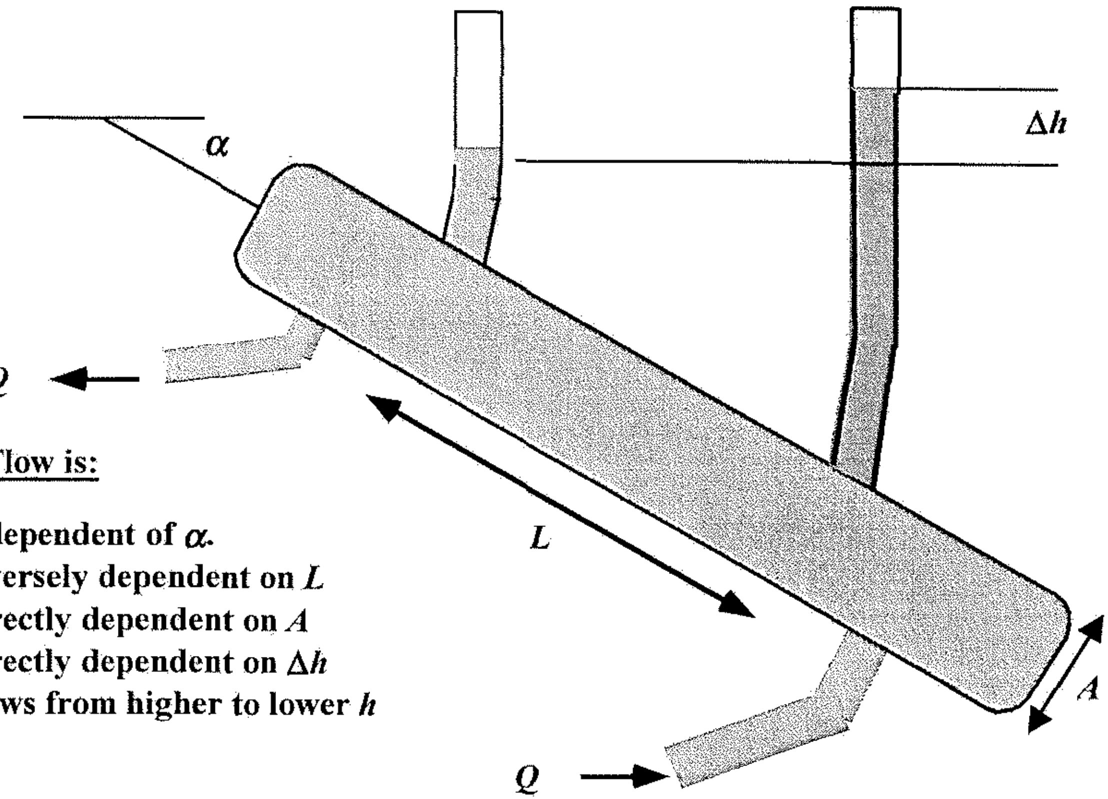
We do not know what theories the Chinese had regarding subsurface water. The Greeks gave a considerable amount of thought to groundwater, and believed that all rivers were fed by springs flowing out of underground caverns. In arid countries like Greece, it is easy to see how rainfall would seem inadequate to supply river flows. Add to this the observation of impressive springs and a few caves, and it is easy to see how such ideas could develop. The Greeks recognized that most of the world's water must be in the oceans. The scientific questions of the day were: How was water pumped from the ocean to the highland springs, and how was the salt removed?

The Greeks missed the following: (1) the Earth does not contain a network of interior caverns, (2) evaporation is the pump that lifts and desalinates seawater, and (3) rainfall is adequate to explain spring and river discharges. The first to quantitatively demonstrate this last critical point were Pierre Perrault and E. Mariotte. In the mid-1600's they proved that rainfall was sufficient to feed the Seine River near Paris. Ocean water recharged the river as the Greeks believed, but the source of the recharge was rain.

It was Henry Darcy in 1856 who discovered the physical laws that govern groundwater flow and successfully applied them to the flow of water in aquifers (the term used by hydrologists for permeable parts of the subsurface). Darcy was an engineer charged with developing a water supply for Dijon, France. He carried out experiments with sand filters, and presented the mathematical underpinning of porous (amount of empty space a substance contains) media fluid flow in an appendix to his engineering report. The great beauty, then and now, of Darcy's approach is that his theoretical formulation can be directly applied to the subsurface flow that occurs in nature.

A simple experiment with a glass cylinder filled with sand captures the essence of Darcy's experiments and Darcy's Law. If water is passed through the cylinder sketched in Figure 11.2, the difference in fluid level in the two taps is directly dependent on the flow rate per unit cross sectional area though the cylinder, directly dependent on the distance between the two taps, and <u>independent</u> of the orientation of the cylinder.

Darcy's Law Relates Water Level Changes in Wells to the Groundwater Flow Rate



The Flow is:

- **Independent of α.**
- **Inversely dependent on L**
- **Directly dependent on A**
- **Directly dependent on Δh**
- **Flows from higher to lower h**

Figure 11.2

These relations may be expressed mathematically:

$$(1) \qquad V = \frac{Q}{A} = -K\frac{\Delta h}{L} = -K\nabla h = -\nabla\Phi$$

where:

Q = Flow rate through the cylinder in cm^3/sec.
A = Cross sectional area of the cylinder.
V = Darcy flux (in cm^3 water per cm^2 cross sectional area per second).
K = Hydraulic conductivity of the media in cm/sec.
h = Elevation (above sea level) of water in the tube taps.
∇ = Gradient operator.

.The symbol Φ assumes K is constant everywhere and provides a very simple formulation that we shall use in Chapter 14. Since ∇h is dimensionless, the Darcy Flux has the units of K. There are several traditional units for K such as gallons/ft^2-day. The table on the next page gives simple metric units and the range in permeability (property by which solids allow fluids to pass) usually found in rocks and sediments. Notice that hydraulic conductivity of natural minerals ranges over ~13 orders of magnitude! It is important to realize, and therefore we emphasize here

Typical Values and Units of Hydraulic Conductivity (K)

from Freeze and Cherry, Groundwater, Prentice Hall, 1979

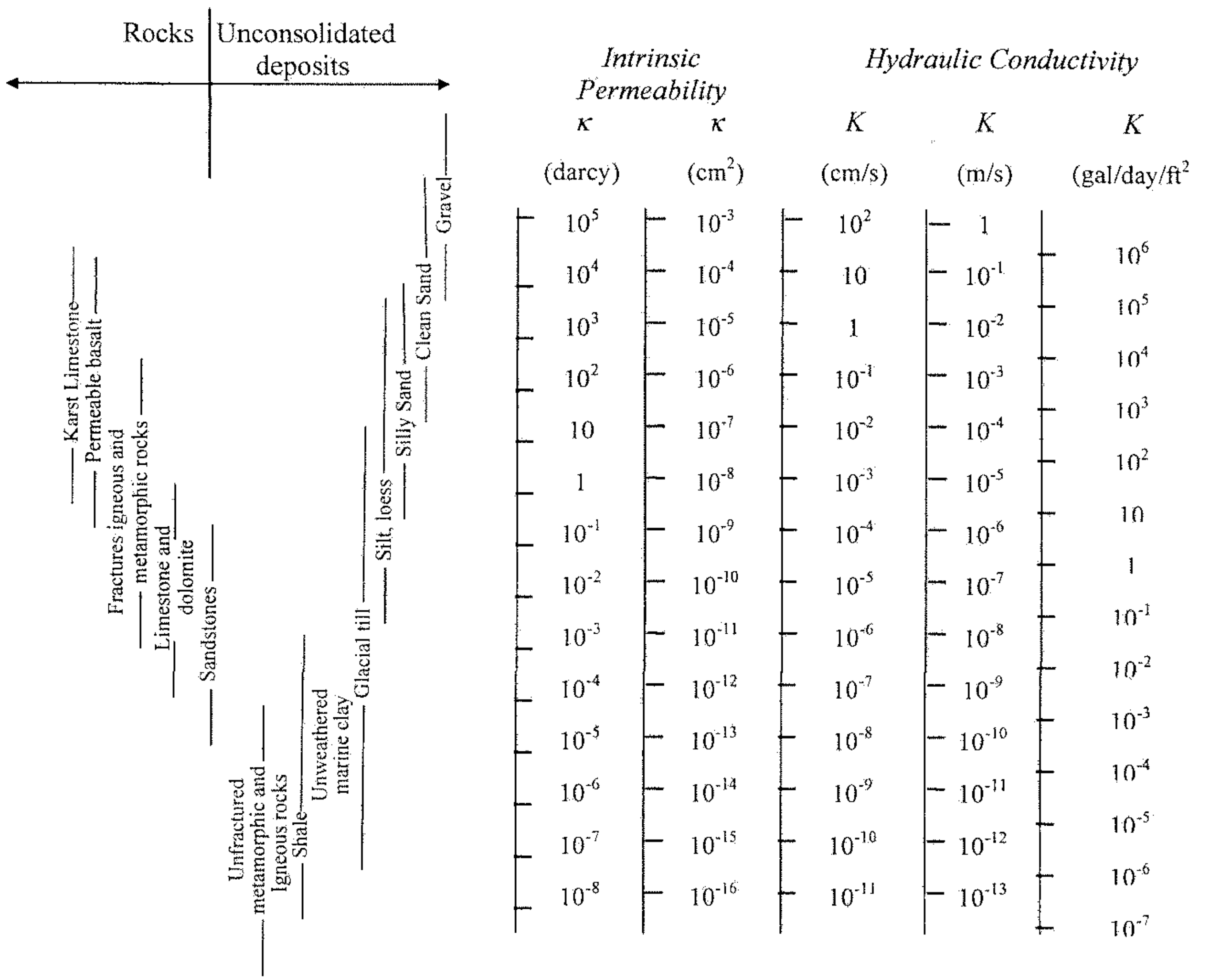


Table 2.3 Conversion Factors for Permeability and Hydraulic Conductivity Units

	Permeability, κ*			Hydraulic conductivity, *K*		
	cm²	ft²	darcy	m/s	ft/s	gal/day/ft²
cm²	1	1.08×10^{-3}	1.01×10^{3}	9.80×10^{2}	3.22 x 103	1.85×10^{9}
ft²	9.29×10^{2}	1	9.42×10^{10}	9.11×10^{5}	2.99 x 106	1.71×10^{12}
darcy	9.87×10^{-9}	1.06×10^{-11}	1	9.66×10^{-6}	3.17 x 10-5	1.82×10^{1}
m/s	1.02×10^{-3}	1.10×10^{-6}	1.04×10^{5}	1	3.28	2.12×10^{6}
ft/s	3.11×10^{-4}	3.35×10^{-7}	3.15×10^{4}	3.05×10^{-1}	1	5.74×10^{5}
gal/day/ft²	5.42×10^{-10}	5.83×10^{-13}	5.49×10^{-2}	4.72×10^{-7}	1.74 x 10-6	1

*To obtain κ in ft², multiply κ in cm² by 1.08×10^{-3}

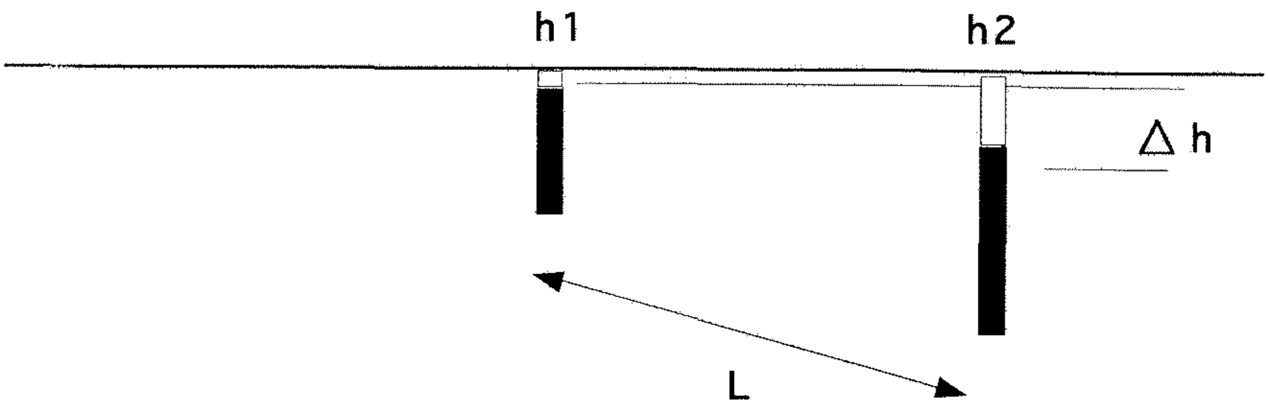
Figure 12.1 adapted from Freese & Cherry, 1979 again, that although V has the dimensions of velocity (cm/sec) and for this reason is often referred to as the Darcy or superficial velocity, it is actually a flux - the flux of water through a surface perpendicular to the flow (measured by m^3 of water passing the surface per unit area of the surface per unit of time).

The true velocity of the ground water v is given by the Darcy flux divided by the fraction of the porous media occupied by moving water. In sand, this fraction is the water filled porosity ϕ_w. In a fractured igneous rock it is the fracture porosity, ϕ_{wf}

$$(2) \qquad \text{True fluid velocity } v = \frac{V}{\phi_w} \text{ or } \frac{V}{\phi_{wf}}.$$

Darcy's Law can be directly applied to the natural environment. Consider the aquifer in the diagram below (Figure 11.3). If the water level in wells is as indicated, the Darcy flux through the aquifer is easily calculated to be 10 gal/ft^2-day or 1.5 m/yr. Notice that the flow direction is indicated by the difference in water level in the wells. If the downdip well had a higher water level than the updip well, the flow would be out of rather than into the aquifer.

Geologic Flow Can Be Directly Determined From Darcy's Experiment



$\Delta h = h_1 - h_2 = 53'$
$L = 1$ mile
$\Delta h/L = .01$
$V = -K\, dh/dl = .01K$

$K = 10$ gal/ft^2-d or $K = 148$ m/yr
$V = 0.1$ gal/ft^2-d $V = 1.5$ m/yr

Figure 11.3 Darcy's Experiment

Darcy's Law also applies to the unsaturated zone above the water table. In this case, the hydraulic conductivity is a function of water saturation, S_w. The total water in the porous media is $\phi_w S_w$. As the water saturation decreases, the hydraulic conductivity decreases. At a certain saturation, called the irreducible water saturation (S_{iw}), water will no longer gravitationally drain from the soil. Thus a short time after a rain has charged the soil with water and the gravitational water has had time to drain to the water table, the soil will be filled to its field capacity, $FC = \phi_w S_{iw}$. Typically: $S_{iw} \sim 0.25$, $\phi_w \sim 0.40$, and $FC = 0.1$. Plants, utilizing capillary forces, are able to pull non-gravitational water out of the soil. They cease to be able to do this when they have developed a negative pressure of about 1.5 MPa (or 15 atmospheres, 1 atmosphere = 1.013×10^5 Pa). At this point, they wilt; this is known as the wilting point stage.

How long will pesticides take to reach the ground water table directly under a field? The answer is a surprisingly long time. Roughly speaking, to reach the ground water table, water containing pesticides must replace the non-gravitational water in the soil between the surface and the ground water table. If the yearly infiltration in an area is 0.25 m of water per year (e.g., a flux into the ground of 0.25 m^3/m^2-yr), $FC = 0.1$, and the soil layer is 10 m thick, there is 1 m^3 of non-gravitational water between the surface and the ground water table per m^2 surface area of the field. Consequently, it will take 4 years for the non-gravitational water to be replaced. It will take even longer for the water to migrate from under the field to water wells or rivers where it may be detected. For example, if the area of ground water recharge (water infiltrating the subsurface) is approximately equal to the area of discharge (where water are produced) and the subsurface water moves horizontally, water will move at a Darcy velocity equal to the infiltration rate. If the porosity of the water-saturated soil below the water table is 10%, the true velocity for an infiltration rate of 0.25 m/yr will be 2.5 m/yr. Thus if the field is 500 m from the nearest well or stream, the contamination will take 200 years to arrive. The biggest problem with ground water contamination is the long delays involved. If you remember nothing else from these notes, please remember this.

Because so much shear takes place between a moving fluid and a solid matrix, the subsurface is a very effective mixer, a somewhat slowly operating natural blender. A useful analogue for contaminant transport is thus a well-stirred tank (Figure 11.4):

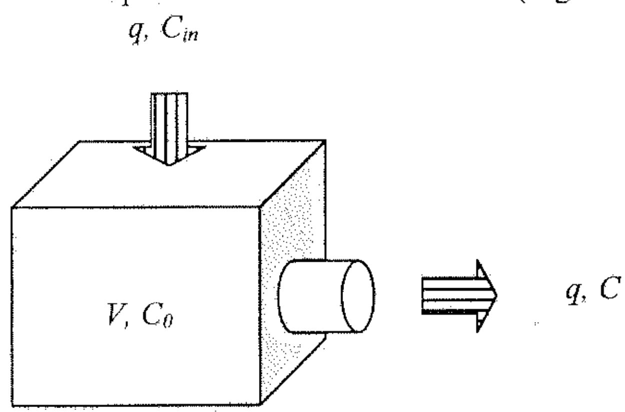


Figure 11.4 A well-stirred tank as an analogy for contaminant transport

If the flows into and out of the tank are equal and indicated by q, the volume of the tank in compatible units is V, and the tank is initially filled with water with contaminant concentration C_0, we can solve for the concentration in the tank as a function of time if we, at $t = 0$, start pouring in water with contaminant C_{in}. We do this by requiring contaminant mass balance. The contaminant we put into the tank either exits or increases the amount of contaminant in the tank:

(3) $$q\,(C_{in} - C)\,\Delta t = V\Delta C.$$

Here ΔC is the change in contaminant concentration in the tank over time increment Δt. If we define a new variable $C' = C_{in} - C$, since C_{in} is by hypothesis constant, $\Delta C = -\Delta C'$, then (3) becomes:

(4) $$qC'\Delta t = -V\Delta C'.$$

This equation may be integrated from $t = 0$ to t and from $C' = C'_0$ to C':

(5) $$\int_0^t \frac{-q}{V}\,dt = \int_{C'_0}^{C'} \frac{dC'}{C'}$$

with the result that:

(6) $$\frac{-qt}{V} = \ln C' - \ln C'_0$$

If $C'(t = 0) = 0$, $C'_0 = C_{in}$, and the solution is:

(7) $$C(t) = C_{in}\left(1 - e^{-qt/V}\right) = C_{in}\left(1 - e^{-t/\tau}\right)$$

where we have defined a residence time $\tau = V/q$

The change in concentration with time is controlled by the residence time, which is just the time required for the balanced inflow and outflow to replace the water in the tank once. However, we can now address the question of how much contamination will result if after a time T, the harmful effects of contaminant application are recognized and the use of the contaminant is stopped. We can calculate this case easily because the system is linear. If C_{in} doubles, $C(t)$ doubles. Thus the effects of application from $T = 0$ to T is the same as an application from $t = 0$ to ∞ followed by application of $(-C_{in})$ from $t = T$ to ∞.

The solution is thus:

$$C(t) = C_{in}\{1 - \exp(-t/\tau)\} \qquad \text{from } t = 0 \text{ to } T$$
$$C(T) = C_{in}\{1 - \exp(-T/\tau)\} \qquad \text{at } t = T$$
$$C(t) = C_{in}\{1 - \exp(-t/\tau)) - C_{in}\{1 - \exp(-(t-T)/\tau)) \qquad \text{from } t = T \text{ to } \infty.$$

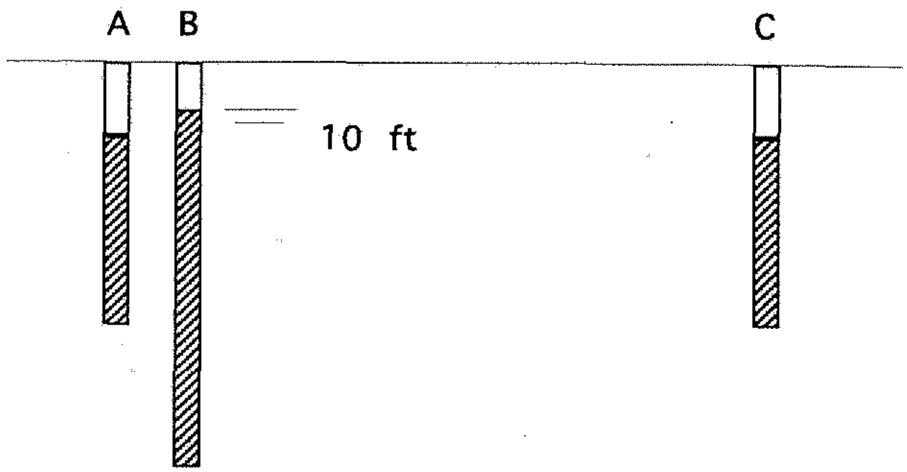
The effluent concentration is maximum at $t = T$, and it is easy to see that a contaminant problem becomes serious as the application time becomes comparable to the residence time of water in the subsurface. For example, if the residence time of the fluid is 1000 years and contaminant is applied for 100 years, the maximum discharge concentration is $C_{in}\{1 - \exp(-0.1)\} = 0.095\ C_{in}$.

The mixing relation derived above is not realistic. A better approximation might be a series of well-stirred tanks. Clays can adsorb contaminant, greatly increasing the effective volume of the tank (contaminant capacity of the subsurface), and delaying the development of a contaminant problem. On the other hand, fractures can short circuit water flow and greatly decrease the volume of the tank. The advantage of the above formulation is that it provides insights into the general nature of the problem in a simple way.

Problem Set 11: Subsurface Fluid Flow

11.1. Assuming the hydraulic conductivity of the soil with 10% porosity is 10^{-5} m/sec, calculate the fluid flux through a 12 m thick aquifer if the gradient in the free standing head in wells drilled into the aquifer is 20 m per kilometer. Assuming a contaminant does not react with the sediment, how long will it take the contaminant to move a distance of 10 km through the aquifer?

11.2 If the water in two wells located very near one another is as indicated in the diagram below, with a well further away with the same depth as a well having the same standing water level as A, what is the direction of water movement? Assume that the wells are open mainly at their lower portions. What further piece of information would you need to calculate the water velocity?



11.3 If the ground water table is 30 ft below the surface, the undrained (e.g., non-gravitational) water content of the soil is 10%, and the yearly infiltration 15 cm per year, estimate the time required for a pesticide to reach the ground water table.

11.4 Show that equation (7) follows from equation (6).

Chapter 12. Regional Ground Water Flow

The water table follows topography and intersects the land surface at lakes and rivers. Rainfall and infiltration in upland areas raise the elevation of the water table under hills until the Darcy flow to the rivers is sufficient to remove the water that infiltrates (the system is then at steady state). We can learn a great deal about subsurface flow if we analyze, as we did in the case of glacial rebound, a simple case where the groundwater table is perturbed cosinusoidally (Figure 12.1). In many cases, this is a fairly good approximation of the natural situation:

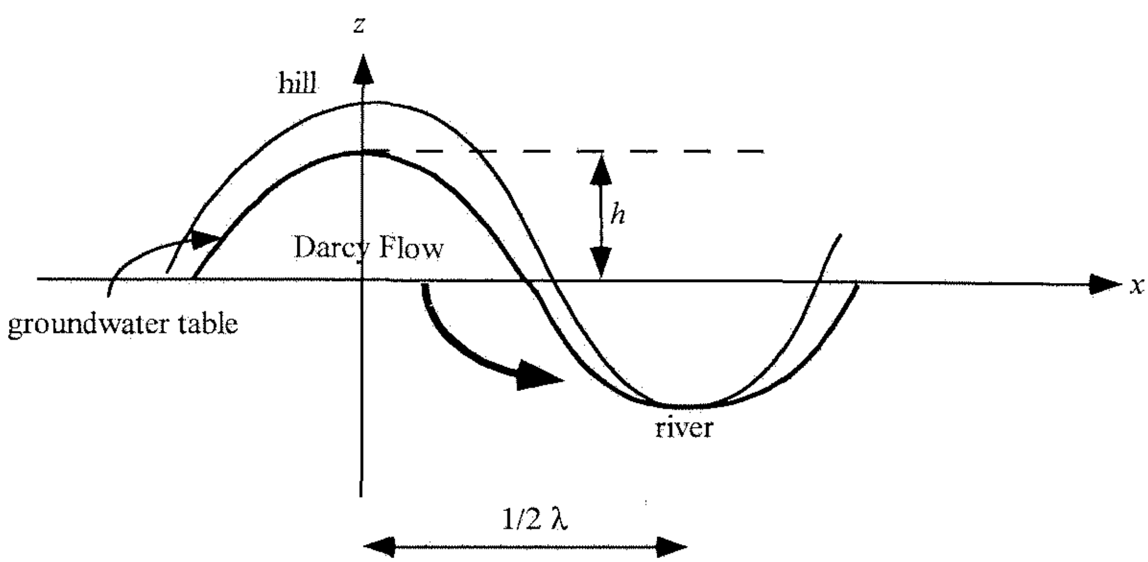


Figure 12.1

Here h is the elevation of the water table above the x-axis (the elevation of the water table with respect to the river surface would be $2h$), and λ is the wavelength of the cosinusoidal curve representing the topography ($\lambda = 2\pi/k$, where k is the wavenumber). The elevation of the groundwater table, $h(x,y,z)$, under these conditions is mathematically described by $h = h_0 \cos kx$. We can solve for the flow of subsurface water by combining Darcy's Law (conservation of momentum) and conservation of mass:

(1) Darcy's Law

$$V(x,y,z) = -K\,\nabla h = -(K\frac{\partial h}{\partial x}, K\frac{\partial h}{\partial y}, K\frac{\partial h}{\partial z})$$

(2) Conservation of Mass

$$\nabla \cdot V = -(K\frac{\partial h}{\partial x} + K\frac{\partial h}{\partial y} + K\frac{\partial h}{\partial z}) = 0$$

Which, for constant K, combine to give Laplace's equation:

$$(3) \qquad \nabla^2 h = 0 .$$

Letting $h(x, z = 0) = h_0 \cos kx$ (where $k = 2\pi/\lambda$ and we choose $z = 0$ to be at the surface). We also let $h(x, z = -\infty) = 0$ (where we choose $-\infty$ to be a great depth).

Solving (3) by separation of variables:

$$h = X(x)Z(z)$$

$$\nabla^2 h = \frac{1}{Z}\frac{\partial^2 Z}{\partial z^2} + \frac{1}{X}\frac{\partial^2 X}{\partial X^2}$$

Setting the first term on the right = k^2 and the second term on the right = $-k^2$, the solution of each of these new equations is:

$$Z = Ce^{-kz} + De^{kz}$$

$$X = A \cos kx + B \sin kx.$$

Applying the boundary conditions:

$C = 0$ because the solution is zero at great depth or large $(-z)$ value.

$B = 0$ because the solution must be a cosine solution and not sine.

Therefore the solution is:

$$(4)\ h = h_0 e^{kz} \cos kx .$$

Here the wave number $k = 2\pi / \lambda$, and h_0 is the amplitude of the cosinusoidally perturbed ground water table.

By Darcy's law (equation 1 above):

$$(5)\ V_z = -K\frac{\partial h}{\partial z} = -Kh_0 ke^{kz} \cos kx = -V_{z_0} e^{kz} \cos kx$$

$$(6)\ V_x = -K\frac{\partial h}{\partial x} = Kh_0 ke^{kz} \sin kx = V_{z_0} e^{kz} \sin kx .$$

Where:

K(in cm/sec) = hydraulic conductivity
h_0(in cm) = maximum initial height of the ground water surface above the x-axis
k(in cm^{-1}) = the wavenumber

$V_{z_0} = Kh_0k.$

As in the case of glacial rebound, these fluxes (V_x and V_z) apply at a time t when the water table amplitude is h. If infiltration maintains the water table at a constant elevation (as is usually the case), then the ground water elevation, h_0, and the ground water velocity, V_z and V_x, remain constant. However, as in the glacial rebound case, we can use the fact that the groundwater will fall at a rate of V_z/ϕ (where ϕ is the porosity) to determine how the water table will flatten as a function of time if infiltration ceases. (e.g. if there were a severe drought).

From (5) we have:

$$(7) \qquad \frac{V_z(x,z=0)}{\phi} = \frac{-Kh_0k\cos kx}{\phi} = \frac{-V_{z_0}\cos kx}{\phi}$$

also, the velocity can expressed as: $\frac{dh}{dt}\cos kx$.

Hence, $\frac{dh}{dt}\cos kx = \frac{-Kh_0k\cos kx}{\phi}$ and,

$$\int_0^t \frac{-Kk\,dt}{\phi} = \int_{h_0}^{h} \frac{dh}{h} \text{ and,}$$

$$\ln\frac{h}{h_0} = -\frac{Kkt}{\phi}$$

$$(8) \qquad h = h_0e^{-t/\tau} \quad \text{where, } \tau = \frac{\phi}{Kk}$$

Because of the linearity of Darcy's Law, more complicated solutions can be obtained by superimposing simple cosinusoidal solutions of different wavelengths and phases by adding the solutions of the simpler cases together.

Problem Set 12:

12.1 Verify (4) is a solution of (3) -- substitute it into (3).

12.2 Where does the $\frac{dh}{dt}\cos kx$ term come from in equation (7)?

12.3 Show that (8) follows form (7).

12.4 Suppose the lowest point of a cosinusoidal groundwater table is at the elevation of a local stream and the highest point is under the crest of a hill running parallel to the stream 7 km away. If the hydraulic conductivity (K) is 4×10^{-8} m/sec, and the infiltration rate at the hillcrest is 25 cm/yr, what is the height of the water table under the hill relative to the stream?

12.5 What will be the elevation of the water table in problem 12.3 if it does not rain for 10 years? At what locations would residents need to worry least about their wells drying up?

12.6 Estimate, assuming flow travels 7 km, the residence time of water in the system in Problem 12.4 (from precipitation at the crest of the hill to arrival at the stream).

Chapter 13: Groundwater Flow and Man

Water resources management must deal with the complexities of regional flow (Figure 13.1) and many local sources and sinks of water. Predicting which direction a contaminant will migrate, and when it might arrive at a particular point is not always easy. Nevertheless, there are simple schemes for roughly calculating ground water movement and contaminant migration.

In this chapter, we use the linear nature of Darcy's Law to derive a simple method of subsurface flow analysis that is particularly useful in analyzing man's impact on subsurface flow. We assume the hydraulic conductivity of the subsurface is constant and the same everywhere. In this case, Darcy's Law states that the Darcy's Flux equals the negative gradient of a flow potential: $V = -\nabla\Phi$. The regional flow potentials and flow to and from point sources (wells) are easily derived.

I. Regional Flow

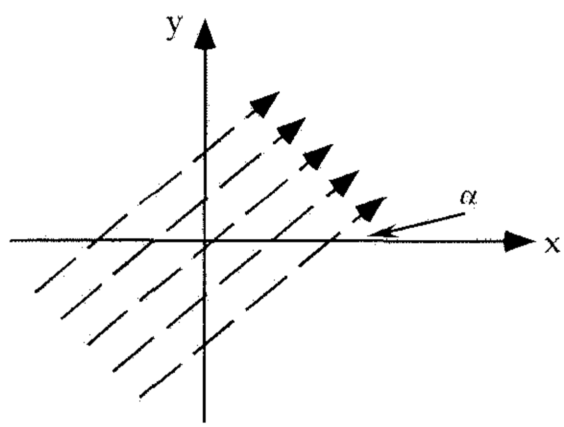


Figure 13.1

If the regional flow is to the northeast at an angle α, it is straightforward to deduce:

$$V_x = V\cos\alpha = -\frac{\partial\Phi}{\partial x}$$

$$V_y = V\sin\alpha = -\frac{\partial\Phi}{\partial y}$$

Integrating these expressions we find that:

(1) $\Phi_{Regional}$ = regional flow potential $= -V(x\cos\alpha + y\sin\alpha) + C$.

I. Flow into (or out of) a well

At any distance r from the well, the radial Darcy flux must equal the total flow from the well, Q, divided by the area across which that flow is taking place (Figure 13.2):

$$V_r = -\frac{\partial \Phi}{\partial r} = -\frac{Q}{2\pi r b}$$

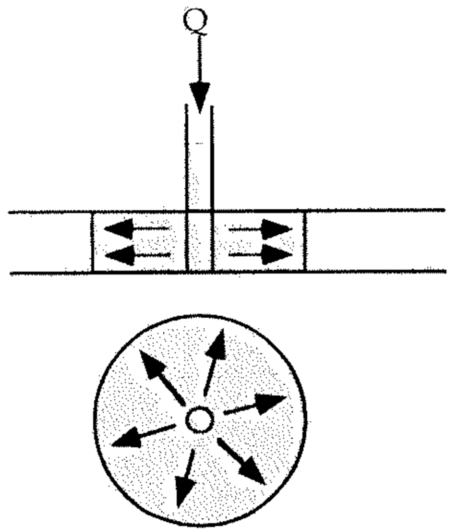


Figure 13.2

Integrating we obtain:

(2) $$\Phi_{\text{well}} = -\frac{Q}{2\pi b} \ln r + C.$$

where:

$$r = \sqrt{(x - x_i)^2 + (y - y_i)^2}$$

$$\Phi_{well} = \frac{-Q}{2\pi b} \ln\left[(x - x_i)^2 + (y - y_i)^2\right]^{0.5} + C$$

The linearity of Darcy's Law allows the flow potentials to be added:

(3) $$\Phi_{total} = \Phi_{regional} + \sum_{i=1}^{n} \Phi_{well}$$

So, for a region in which production and injection wells are superimposed on regional flow:

$$\Phi = -V\left(x \cos \alpha + y \sin \alpha\right) + \sum_{i=1}^{n} \frac{-Q_i}{2\pi b} \ln\left((x-x_i)^2 + (y-y_i)^2\right)^{1/2}$$

where $Q > 0$ denotes injection wells. An injection well is one where water is infiltrating into the ground.

Consider flow between an injection and production well with no regional flow (Figure 13.3):

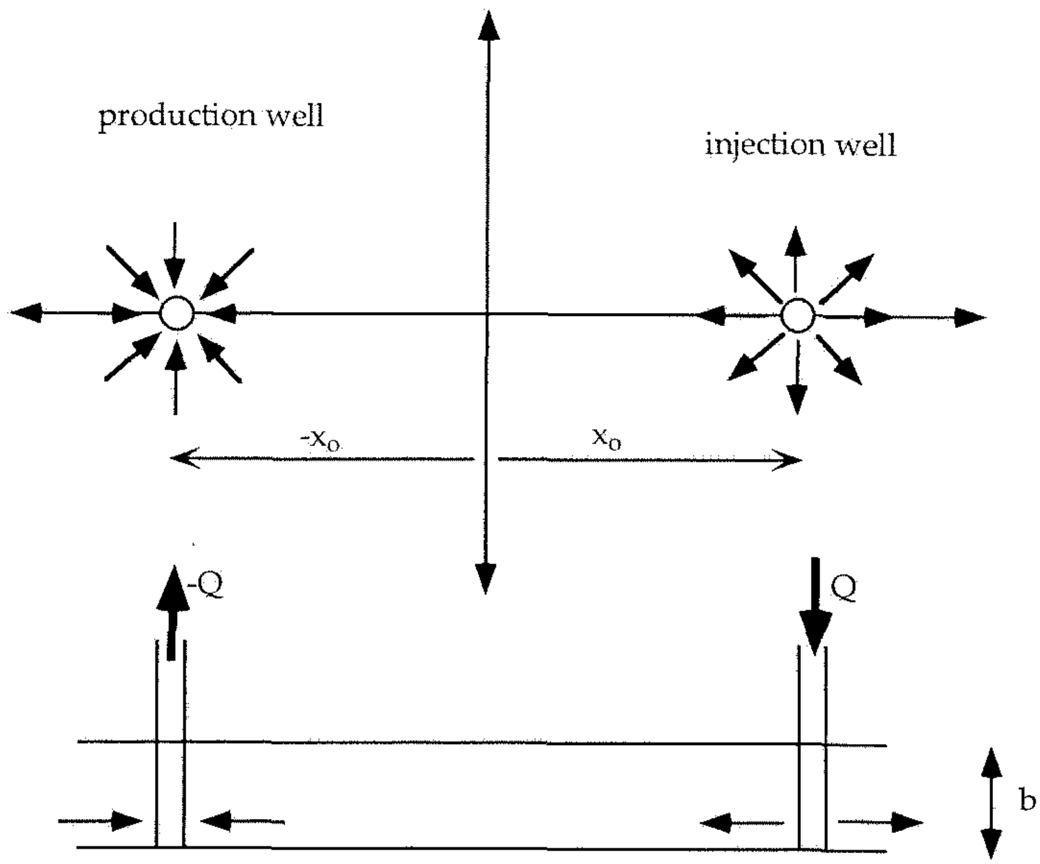


Figure 13.3

where b is the thickness of the aquifer.

$$(4) \qquad \Phi = \frac{-Q}{4\pi b}\left[\ln\left(\left(x+x_0\right)^2 + y^2\right) - \ln\left(\left(x-x_0\right)^2 + y^2\right)\right]$$

Differentiating with respect to x and y gives the flux of the component in each direction:

(5) $$V_x = -\frac{\partial \Phi}{\partial x} = \frac{-Q}{2\pi b}\left[\frac{x+x_o}{(x+x_o)^2 + y^2} - \frac{x-x_o}{(x-x_o)^2 + y^2}\right]$$

(6) $$V_y = -\frac{\partial \Phi}{\partial y} = \frac{-Q}{2\pi b}\left[\frac{y}{(x+x_o)^2 + y^2} - \frac{y}{(x-x_o)^2 + y^2}\right]$$

Since the true velocity (v_x) is V_x/ϕ(where lowercase ϕ is porosity, not potential), we can calculate the time it takes for fluid to travel from the injection to the production well along the x-axis by integrating dx/v_x from $-x_0$ to x_0.

(7) $$T_o = \int_{x_o}^{x_o} \frac{dx}{v_x} = \int_{x_o}^{x_o} \frac{\pi b \phi}{x_o Q}(x^2 - x_o^2) = \frac{\pi b \phi}{x_o Q}\left[\frac{x^3}{3} - x x_o^2\right]_{x_o}^{x_o}$$

(8) $$T_o = \frac{4\pi x_o^2 b \phi}{3Q}$$

Example -- for:

$x_o = 423\text{m}$

$b = 10\text{ m}$

$\phi = 25\%$

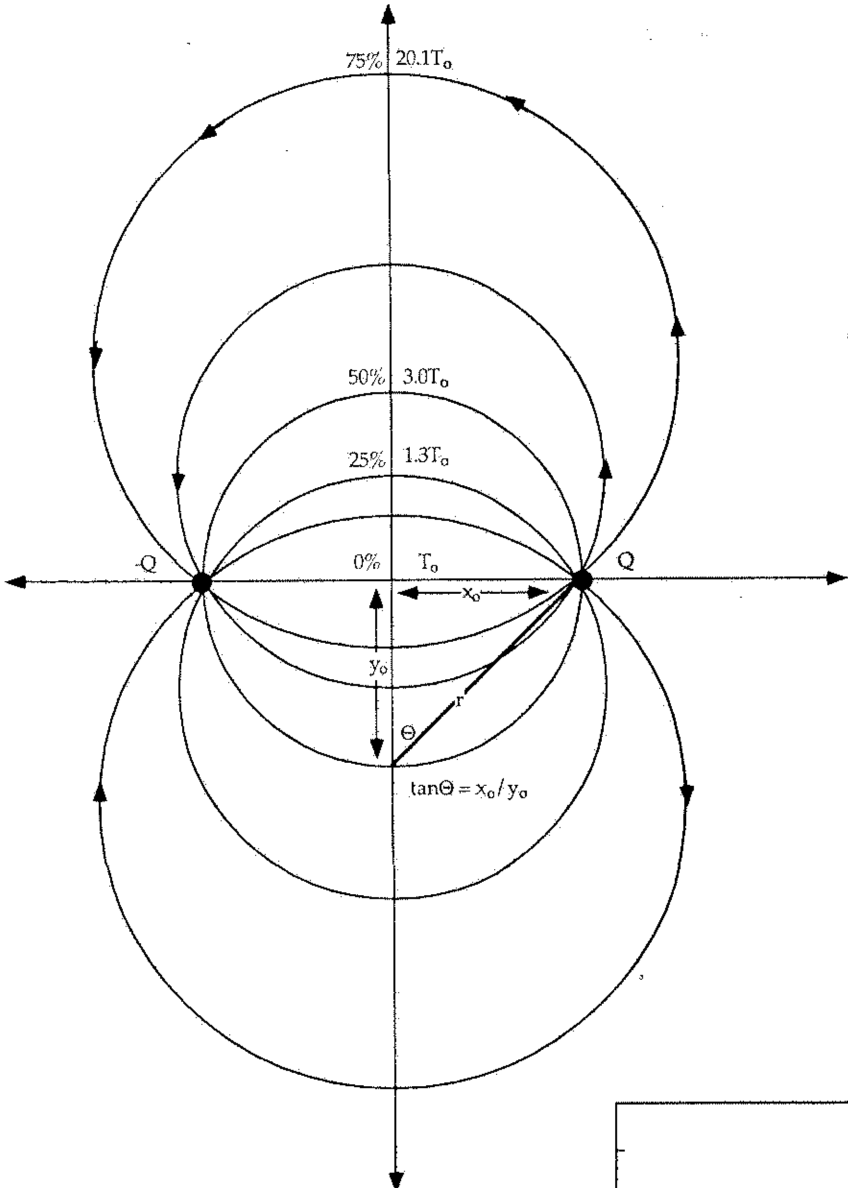
$Q = 50\text{ m}^3/\text{hr}$

$T_o = 4.3$ years

Chasing Streamlines

Equations (5) and (6) give V_x and V_y everywhere so we can easily calculate the transit times for fluids along other streamlines connecting the injection and production wells. For example, we could choose to exit the injection well at N 45°W and simply pick a point a few feet from the well along this trajectory. We could then use (5) and (6) to calculate the x and y velocity of the fluid at this location, step out along this new trajectory a distance $v/\Delta t$, to a new point, calculate V_x and V_y at this point, and step out a distance $v/\Delta t$, etc., keeping track of the total time required for the fluid to reach each point. Since flow out of the injection well will be uniform in all directions, the angles indicate the fraction of total flow. For example, the flow leaving the well between S 45°W and N 45°W will be 25% of the total (see Fig. 13.4). As a result, the times required for different streamlines to arrive, and for different percentages of the flow out of the injection well to be captured by the production well, can be calculated by this kind of iterative integration.

Plotting log T/T_0 vs. the % captured shows that the time to capture successive increments of injected fluid increases faster than exponentially. In the specific case considered above, where T_0 is 4.3 years, 50% of the injected fluid will be captured in 13 years, and 90% capture will require 1212 years (Figure 13.4).


75%
20.1T$_o$
50%
3.0T$_o$
25%
1.3T$_o$
-Q
0%
T$_o$
Q
x$_o$
y$_o$
r
Θ
tanΘ = x$_o$/y$_o$

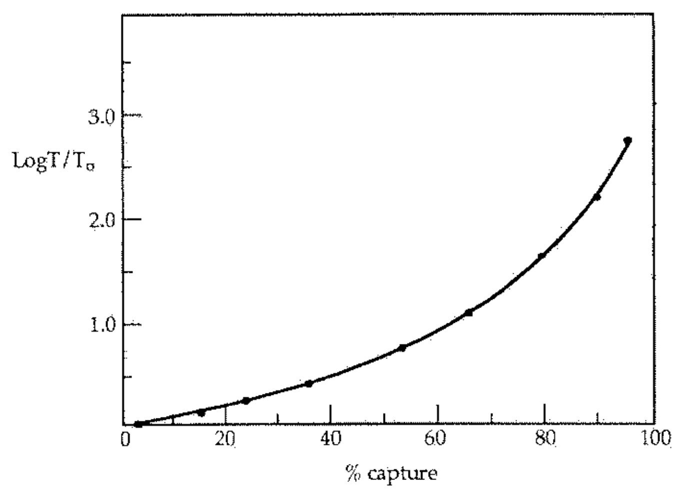

3.0
LogT/T$_o$
2.0
1.0
0
20
40
60
80
100
% capture

Figure 13.4

Figure 13.5 shows the results from 13.4 in a slightly different form. In this figure we contour the age of injected water along streamlines (dashed lines). The figure shows that water injected into the upper well will enter the lower well in $T_0 = \sim 4.3$ years. The age contours on the streamlines pointing away from the producing well will clearly take a very long time to enter the production well.

No Regional Flow

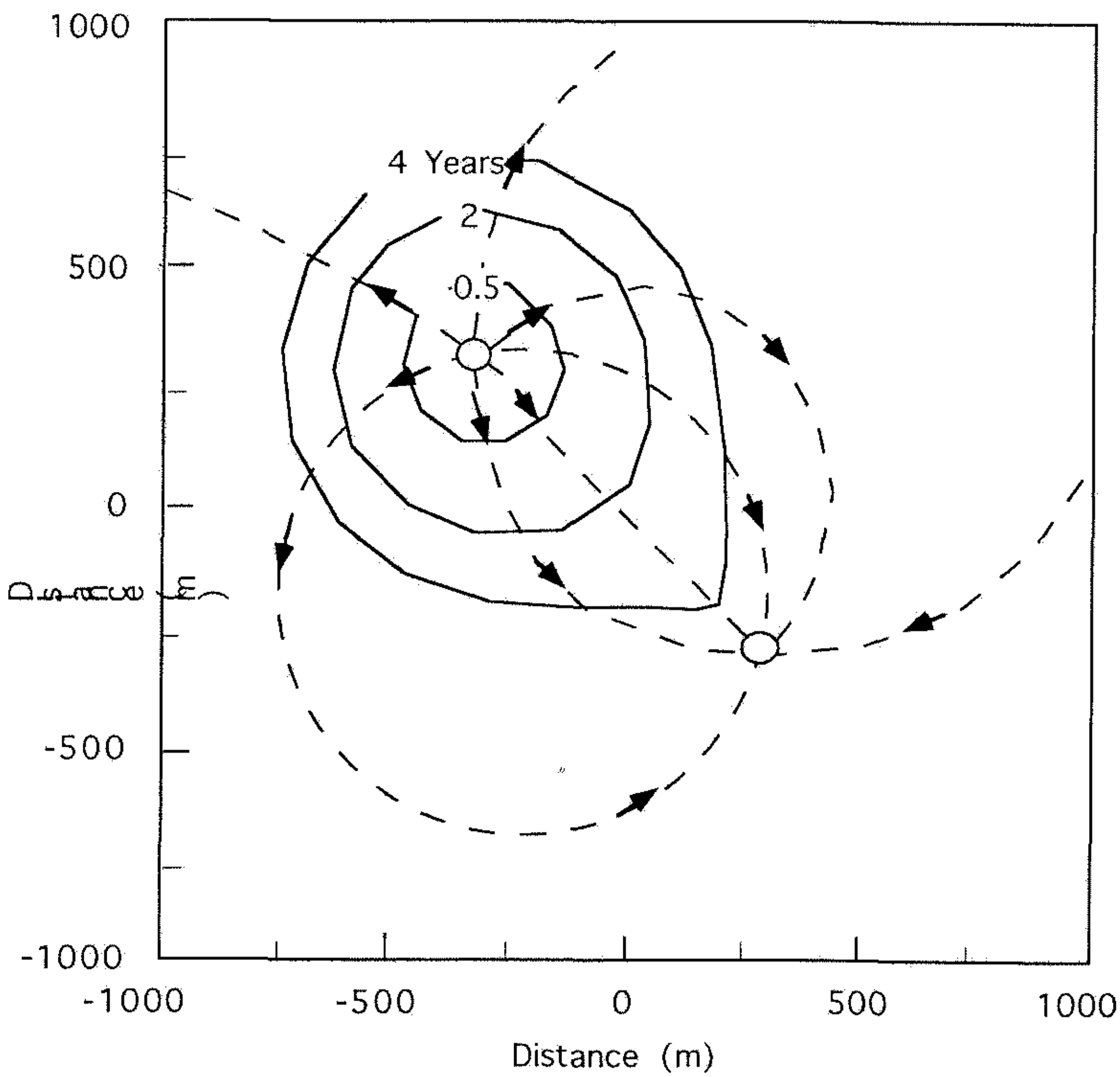


Figure 13.5

If these same injection and production wells lie in an area where there is a regional flow of ground water, the ultimate capture of injection well fluids by the production well will be limited by the fact that some of the flow lines will be swept downstream and never enter the production well. Only some of the streamlines from the injection well will captured, and the total possible recovery is reduced from the non-regional flow case as shown in a plot of % capture vs. time in figure 13.6. In the case shown there only about 35% of the injection well flow will ever be captured by the production (or discharge) well, and this ultimate capture will be attained quickly (~10 years).

Regional flow to NE

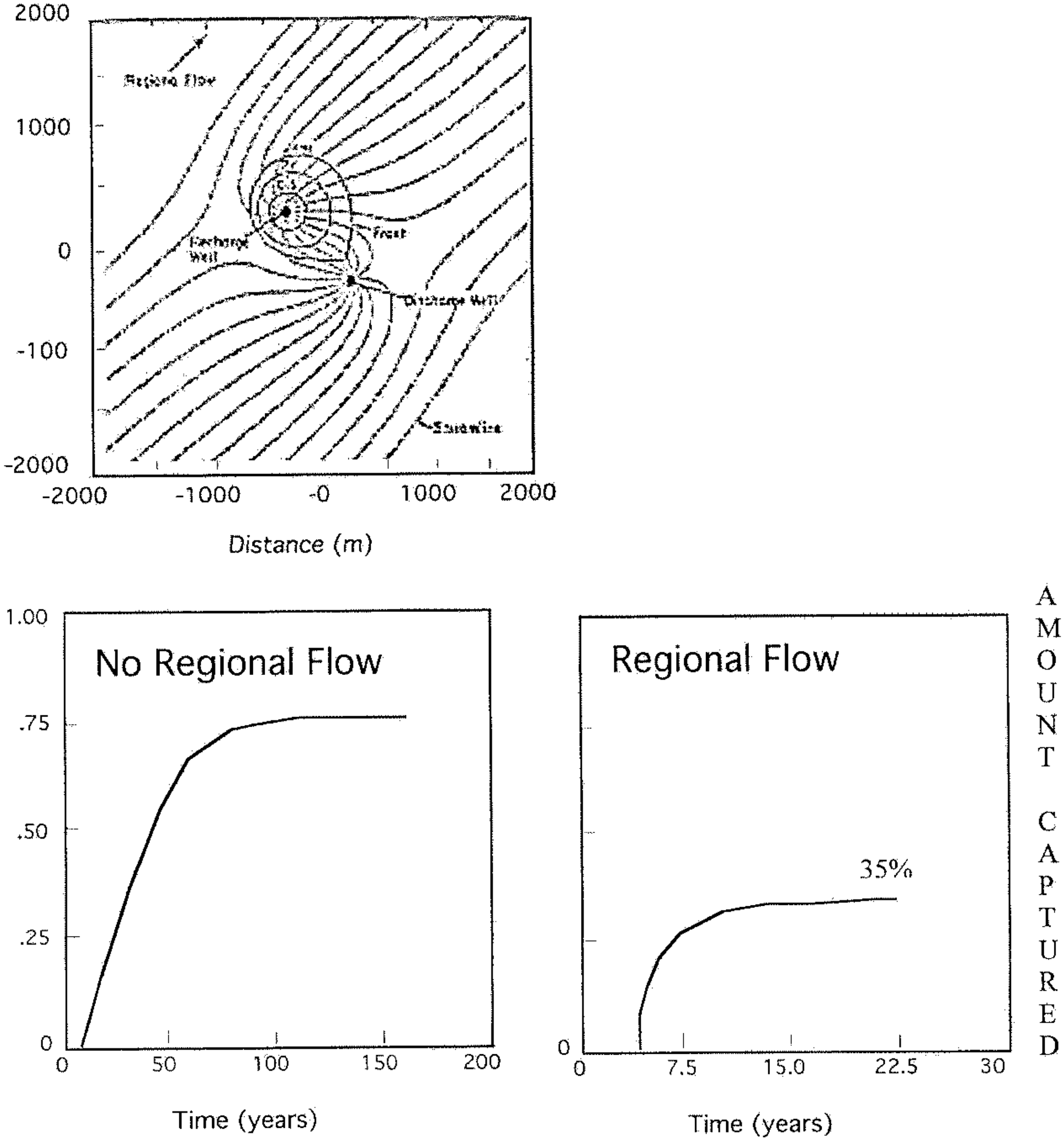


Figure 13.6.

Problem Set 13: Recovering Subsurface Contaminants

13.1 Show that equation (5) and (6) follow from (4) and Darcy's Law.

13.2 Show (8) *follows from* (7).

13.3 Assuming there is no regional flow, how long will it take for a well producing at 20 m^3/hr to capture 75% of the contaminant injected at the same rate into an injection well 100 m away if both wells penetrate a nearly horizontal sandstone aquifer 12 m thick with porosity 25%? What would be some simple ways to capture the injected contaminant more quickly? Which possibility would you most recommend?

Chapter 14: Stratigraphic and Structural Controls

Rivers transport the products of weathering and erosion from upland sources to the sites of deposition in sedimentary basins. Sediment deposition is one of the most important aspects of geology because, as we have seen, it is in the time-ordered stacks of sediments that we learn about the history of our planet, the evolution of life, past catastrophic events, etc. Secondly, in a general sense, the patterns of sedimentation control how fluids move in the subsurface. The most fundamental aspect of subsurface sediment permeability is the great contrast between very low permeability fine-grained units such as shales and very permeable coarse-grained or fractured units such as sands and carbonates. The distribution of sand, shale, faults, and fractures in the subsurface is thus of great importance because it controls how and where fluids will flow. Understanding the distribution of lithologies and the effects of fractures and faults remains an important challenge to the geological sciences today.

The general architecture of subsurface sediments is fractal, and can be understood within the context of river flooding which is also fractal. Fracturing and faulting modify the basic sedimentological framework. We first consider sedimentation, which is largely a matter of common sense, and then turn to the effects of fracturing and faulting

Sedimentation

Streams transport solid material in three ways: as dissolved material, as suspended material, and as bed load. Bed load refers to the fact that the sediments at the bottom of a stream actually move downstream under flood conditions. For this reason, bridge pilings must penetrate bedrock. If they do not, the pilings will move downstream with the bed load when the river floods, and the bridge will collapse. This was the basic cause of the failure of the New York State Throughway (Interstate Route 90) bridge between Albany and Utica a few years ago. Excavation of the bed load to basement for the Brooklyn Bridge pilings was done inside a large wooden chamber. Construction was before the bends was understood, and many of the workers suffered from the decompression effects. Roebling, the chief engineer spent so much time in the chamber that he was permanently disabled and spent the remainder of his life crippled and in constant pain. Bed load transport is difficult to measure but is thought to be about 20% of the suspended load transport. Dissolved load is considerably smaller. Suspended sediment transport is thus the most significant. Figure 14.1 shows that the ability of a river to carry suspended material is approximately proportional to a factor between the square and cube of the river discharge.

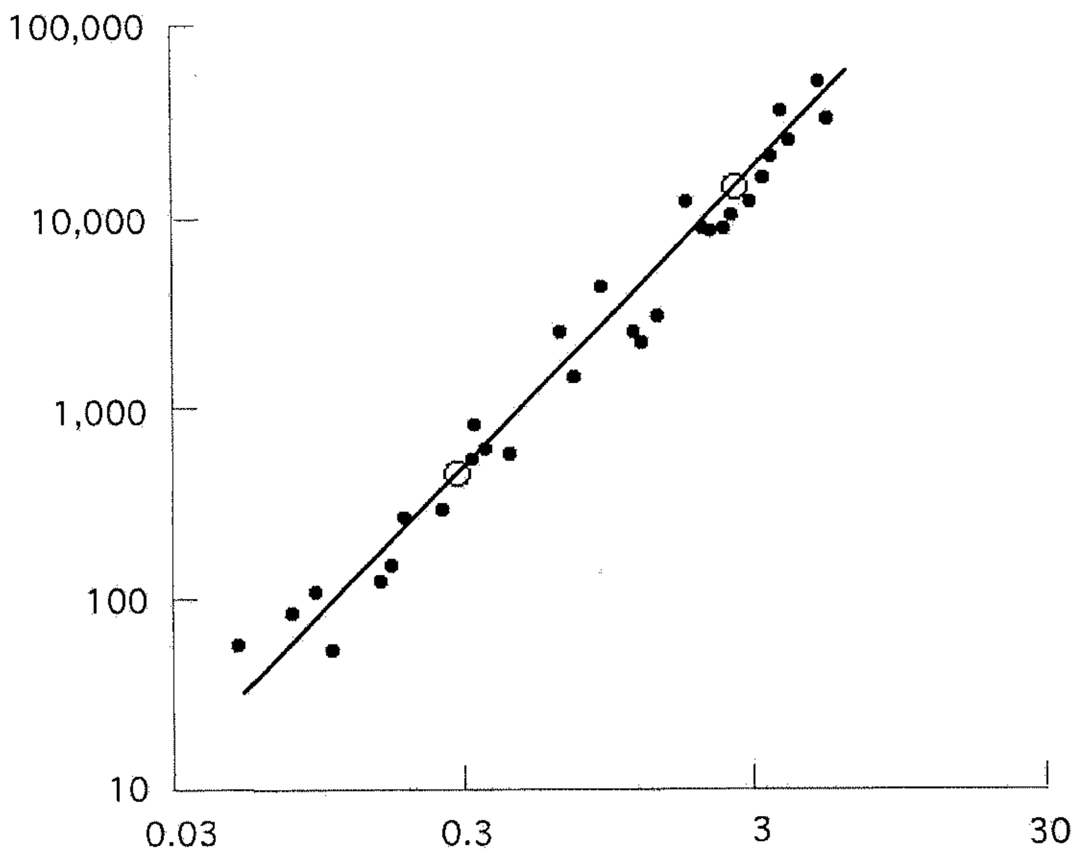


Figure 14.1
The suspended load of a stream (ordinate) increases very rapidly as river discharge (abscissa) increases, as illustrated by this graph based on measurements in the Rio Puerco near Cabezon, New Mexico. (Adapted from Luna B. Leopold and John P. Miller, Ephemeral Streams- Hydraulic factors and their Relation to the Drainage Net, U.S.Geol. Survey Prof. Paper 282-AA, p. 11, 1965).

Turbulent flow in rivers *erodes* at a rate proportional to the velocity of the flow, and carries sediments (within the confines of the stream channel) in proportion to the discharge rate. Sediments are *deposited* where the flow velocity and turbulence are reduced. Thus:

(1) River meanders migrate downstream, leaving behind a sedimentary sequence consisting of channel bottom sands, sand bar deposits, and a silt-clay cap. The reason is that centrifugal forces cause the river velocity to be greatest at the outer parts of a bend and downstream of the bend (Fig. 14.2). Thus, meanders grow in amplitude and migrate downstream with time. Sand bars (called point bars) form on the inner shores of bends, as canoeists know. As the meander migrates downstream, which can happen rapidly (meanders on the Mississippi migrate downstream at 20 m/year), a characteristic sedimentary package is left behind. This package consists of a basaltic bedload gravel, an intermediate interval of point bars, and a cap of fine clay and silt (Fig. 14.2). The fine capping sediments on top of the package are deposited when they overflow their natural levees during flows.

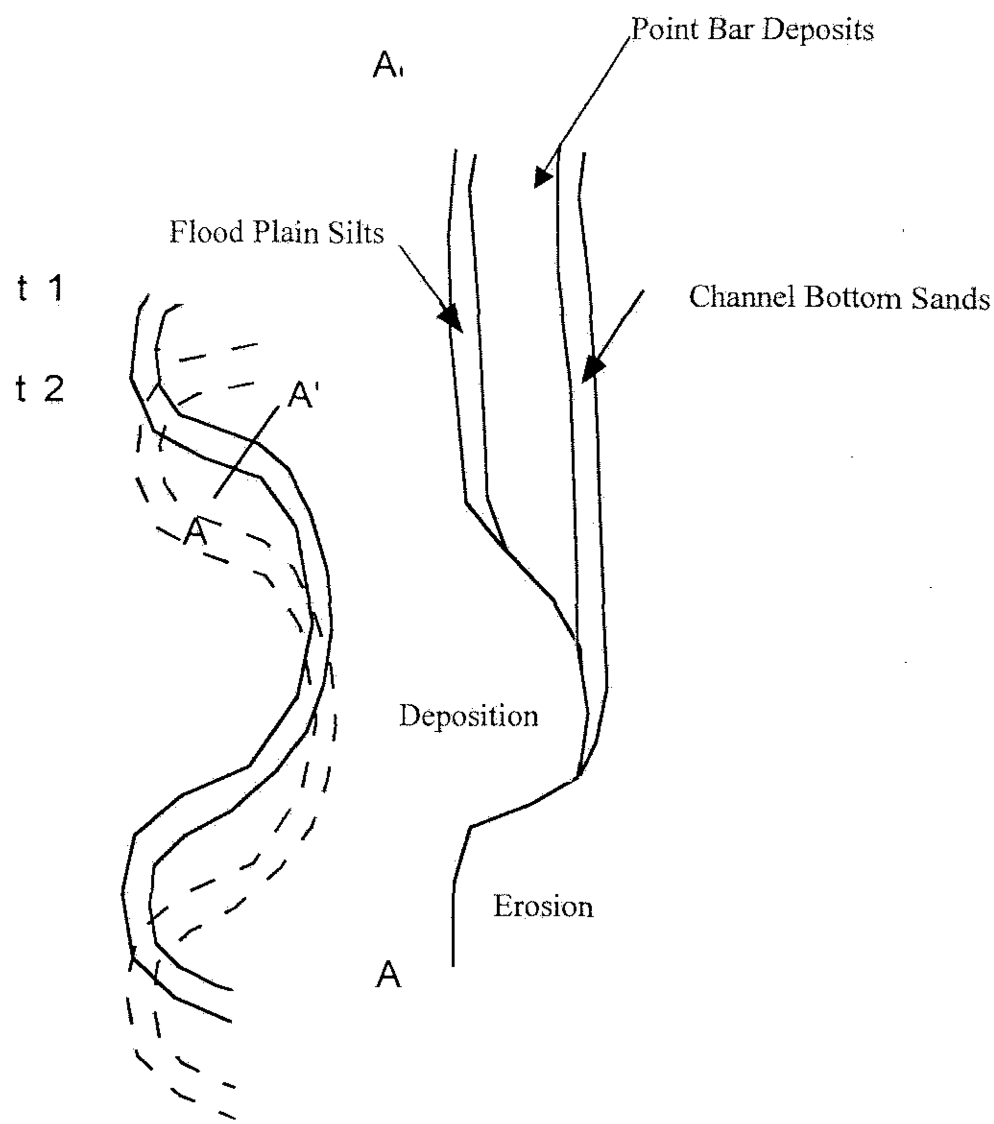


Figure 14.2. River meanders migrate downstream at up to 20 m/yr (Mississippi River). (Adapted from Press and Siever, 1986)

(2) When floods cause water to spill over the riverbanks, the waters spread out (this is what we mean by a flood) and the flow velocity is greatly reduced. Therefore, the suspended sediment load is deposited, the coarser fractions near the banks, and the finer fractions further away (Figure 14.3). The result, over time, is the construction of natural levees that hold back minor floods. *Humans seek to build up these natural levees to avoid flooding.*

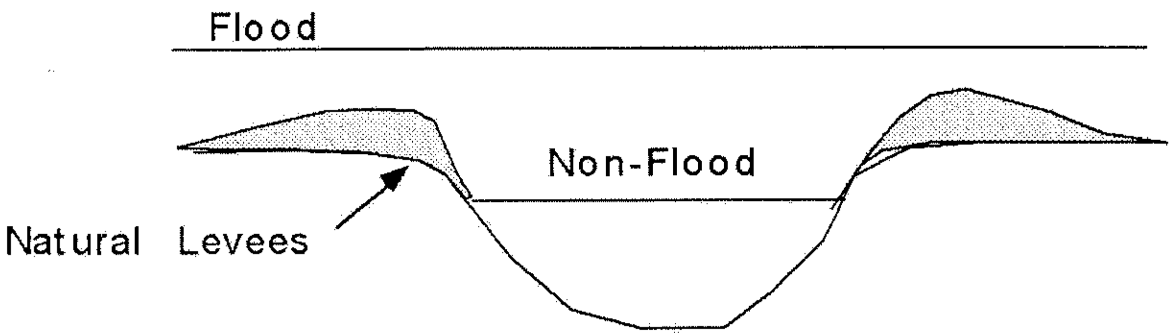


Figure 14.3. Rivers build natural levees during floods as waters spill over the riverbank and the water velocity drops. (Adapted from Press and Siever,1986).

(3) Whether rivers transport more sediment during floods or more during normal discharge depends on the β factor of the river that you derive in the laboratory exercise that is described in the next section. The β factor is the ratio or constant of proportionality between the discharge rates during a 10 year flood, (Denoted Q_{10^1}), to the discharge rates during a 1 year floor, (Denoted Q_{10^0}). A 10 year flood is one that in terms of magnitude of discharge rates - occurs statistically once every 10 years. Similarly, β would be the constant relating the 100 year flood to the 10 year flood, or $\beta = Q_{10^2} / Q_{10^1}$, etc. Since sediment transport rate is proportional to the river discharge raised to the power 2.5 (Fig. 14.1), the ratio of the sediment transport during a 10^n year flow, L_{10^n} to the daily transport rates, $L_{10^{-2.56}}$ is:

$$\frac{L_{10^n}}{L_{10^{-2.56}}} = \left(\frac{Q_{10^n}}{Q_{10^{-2.56}}}\right)^{2.5} = \beta^{2.5\ (2.56+n)}$$

where $Q_{10^{-2.56}}$ is the daily discharge since $1/365 = 10^{-2.56}$.

Assuming the 10^n year flood lasts for D days, the total sediment transported during the flood will exceed the total transported over a year at the daily discharge rates if $D \cdot L_{10^n} > 365 \cdot L_{10^{-2.56}}$ or $\beta^{(2.56+n)\ 2.5} > \frac{365}{D}$. If D = 10 days and β = 2, the sediment transport of the 10^n year flood will exceed a year of daily sediment transport if $n > -0.68$. Thus, the 76 day flood (= 365 x $10^{-0.68}$) will transport more sediments than a year at daily discharge rates if it persists for 10 days. Since all streams flood occasionally, the sediments deposited by streams will be grain-size-layered.

The Hydraulic Conductivity of Layered Sequences

The hydraulic conductivity of a layered sand/shale sequence is nearly that of sand for flow parallel to the strata and very close to the permeability of shale for flow perpendicular to or across the strata. The hydraulic conductivity is thus highly anisotropic - or much greater in one direction than another. This can be easily seen from the following figure and analysis and Figure 14.4:

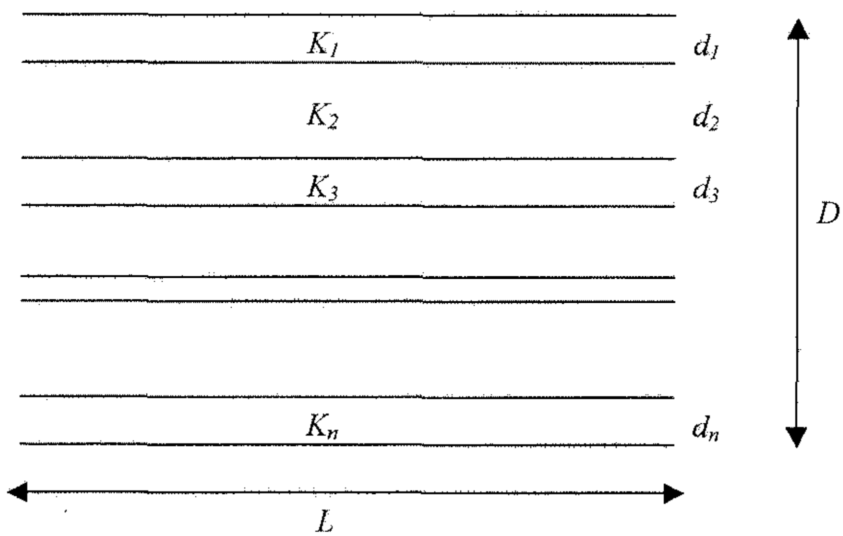


Figure 14.4 The Hydraulic Conductivity of Layered Strata

If water flows vertically, across the stack of sedimentary layers, the Darcy Flux must be the same through each layer. If K_i is the permeability of the ith layer, d_i its thickness, and Δh_i the drop in head across the layer, then:

$$(1) \qquad V_z = \frac{K_1 \Delta h_1}{d_1} = \frac{K_2 \Delta h_2}{d_2} = \frac{K_3 \Delta h_3}{d_3} = \ldots = \frac{K_n \Delta h_n}{d_n}$$

But from an overall point of view,

$$(2) \qquad V_z = \frac{K_z \cdot \Delta h}{D}$$

where $\Delta h = \Delta h_1 + \Delta h_2 + \Delta h_3 + \ldots + \Delta h_n$, and $D = d_1 + d_2 + d_3 + \ldots + d_n$.

From (1) $\Delta h_i = \frac{V_z d_i}{K_i}$, and thus $\Delta h = \sum \Delta h_i = \sum V_z \frac{d_i}{K_i}$. Substituting into (2) and rearranging, we find:

$$(3) \qquad K_z = \frac{d}{\sum_{i=1}^{n} \frac{d_i}{K_i}}$$

If, on the other hand, the flow is parallel to the strata, each strata contributes a fluid flux equal to the head drop Δh across the package in proportion to its permeability, and these flux contributions can be added to get the total flux:

$$(4) \qquad V_x = \sum_{i=1}^{n} \frac{K_i \cdot d_i}{D} \cdot \frac{\Delta h}{L}$$

and the horizontal permeability is thus:

$$(5) \qquad K_x = \sum_{i=1}^{n} \frac{K_i \cdot d_i}{D}$$

It can be seen from (5) that the horizontal permeability of a sand shale package will be nearly that of the sand. From (3), on the other hand it can be seen that the vertical permeability of the package will be nearly that of the shale.

Finally, because the lateral permeability in a grain-size-layered media is so great compared to the vertical permeability, anything that disrupts the lateral continuity of the strata, like faults or fractures, can have a great effect on subsurface flow. Reliably mapping the high-permeability network of the subsurface using geological observations and inference and a host of geophysical techniques is a principle goal of the geological sciences.

Problem Set 14: Sedimentation Systematics

14.1 What is meant by sediments being fractal?

14.2 Show (3) follows from (1) and (2).

14.3 Calculate K_x and K_z if d_i = 5, 10, 7, and 3 cm, $K_{shale} = K_2 = K_4 = 10^{-11}$ m/sec, $K_{sand} = 10^{-5}$ m/sec. What is meant by the statement that K_x is approximately K_{sand}, and K_z is approximately K_{shale}?

14.4 If β=2, how much sediment would a 10 day 1000 year flood deposit compared to 1000 years of sedimentation at the daily discharge rate of the stream?

Laboratory 4: Fractals, Floods, and Sediments

The purpose of this laboratory is to learn how flood frequencies are predicted and to compare the traditional engineering approach to a more geologically based fractal approach.

In the simplest terms, the traditional engineering approach to flood prediction is a procedure that produces a straight line plot of river flooding vs. the frequency of floods greater than specified magnitudes. The straight line can be extended to determine the frequency of flood magnitudes not yet experienced. The abscissa of the plot is the frequency of floods greater than a certain magnitude (in floods per year) determined from a river gauge record of a certain length. If, for example, there has been 1 flood with discharge 5750 m^3/s in a 93 year gauge record, floods of this magnitude have a frequency of 1/93 or 0.011 yr^{-1}. If the second largest flood in the 93 year record was 5570 m^3/s, the frequency of occurrence of a flood this great or larger is 2/93 = 0.022 yr^{-1}. The inverse of the flood frequency is the recurrence rate of a flood as big or bigger than the specified size. In the first case the recurrence is 93 years; in the second case is 46.5 years. A plot of discharge vs. the log of recurrence rate is a straight line for many rivers. On the basis of this line we can say, even though it has not occurred, that the 200 year flood would have a discharge of ~6500 m^3/s. Structures along the river could be designed to survive a 100, 200 or 1000 year flood.

The engineering approach is purely empirical. The application of statistical theory has led to minor modifications. For example, the recurrence rate is calculated for the record length plus one year, and the plot is made on Gimball rather than semi-log paper. The thrust is the same; plot the data in a way that gives a straight line and extrapolate. The modifications, although pleasing theoretically, do not alter the results significantly.

Another approach may be just as good or better. Geologists have known for a long time that many geological processes and features are scale invariant. Without a scale indicator in a photograph, you often cannot tell whether you are looking at something with a scale of feet or a scale of tens or even hundreds of kilometers. For this reason geologists always place their geologic rock hammer or some other suitable scale indicator in their photographs. Mandelbrot showed that the length of a coastline is scale invariant or fractal - the shorter the ruler used to measure a rugged coast, the longer the coastline. A one kilometer length of rocky cost looks exactly the same as a 100 km length of coast if the two are viewed with comparable resolution (e.g., the coast is described by 10 cm segments in the one case and 10 m segments in the second). Fractal synthetic scenery is extremely realistic. Sedimentary layering is fractal in the distribution of thick and thin strata. Earthquakes and faults have fractal distributions.

The underlying cause of scale invariance is self-organized criticality. If energy is poured into a system to the point where it is always on the verge of breaking (like a sand pile to which sand is constantly being applied), the breaks (or slumps) will be scale invariant. The earth appears to be a critical system (or scale invariant) because it is constantly on the verge of breaking. Energy is poured into the earth until, for many phenomena, it is in a state of constant incipient failure, in fractal jargon in a state of self-organized criticality. Scale-invariant or fractal relationships may be the most natural mathematics for describing geologic phenomena. There is presently a great deal of research in geology and engineering in fractal relationships and self-organized criticality (sometimes also called, somewhat inappropriately, "chaos").

A parameter is scale invariant only if the log of the parameter gives a linear plot when plotted against log frequency. For floods to be scale invariant the 10 year flood must be the same factor, β, bigger than the 1 year flood, as the 100 year flood is greater than the 10 year flood, etc. If river discharge is fractal, a plot of the log of the river discharge versus flood frequency should be linear. The slope of the regression will be log β, and the intercept will be the logarithm of the yearly flood, log C. This is illustrated in Fig. 14.5 for a 53 year record from a tide gauge on the Hudson River at North Creek. It can be easily determined from the plot that β = ~1.7, and C=345 m^3/s. Thus the 10 year flood will be ~585 m^3/s and the 100 year flow 1000 m^3/s, etc. The drainage area in this case is 792 mi^2. The curve tends to tail off at high frequencies because events more frequent than 1 year are not resolved accurately in the gauge records.

Figure 14.6 shows the engineering flood prediction plot in which discharge is plotted against log flood frequency. In this plot the 10 year flood is about 660 m^3/s, and the 100 year flood is ~825 m^3/s rather than 1000 m^3/s (the fractal flood prediction). The traditional engineering approach is thus more optimistic than the fractal approach in that less severe flood are predicted.

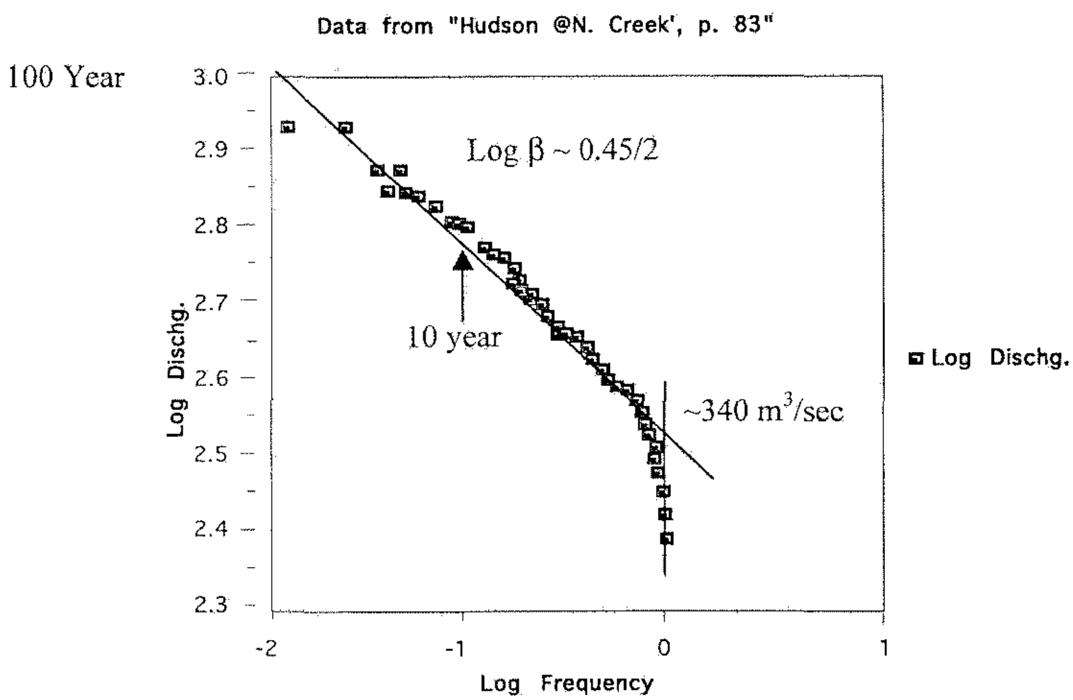


Figure 14.5. Fractal plot of log Discharge v.s. log flood frequency

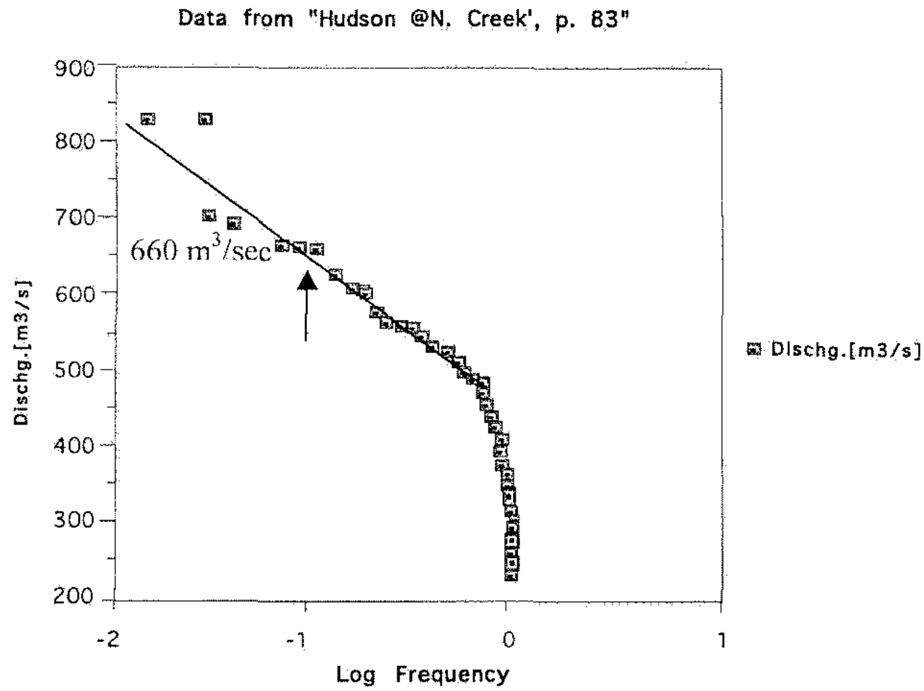


Figure 14.6. Engineering plot of river discharge v.s. log of flood frequency

Exercise:

Data for river gauge stations is readily available in Water Supply papers from the U.S. Geological Survey. Data from a selection of gauges along the Hudson River and other rivers in the northeast is provided. Divide the laboratory into about 5 groups of students.

1. Each <u>student</u> should analyze a gauge station by hand using the methods presented below.

2. Each <u>group of students</u> should analyze one of their gauges using the computers in the lab. Both fractal and traditional engineering plots should be made for this case.

3. The <u>lab group as a whole</u> should then plot the specific discharge, C' and the β factors, versus the catchments area A for all the gauges analyzed

Methods:

The methods are illustrated by the Hudson River at North Creek example. First, consider the computer method of solution. The discharge data is typed into Excel Graph from the Water Supply 1672 from the table on page 83. The discharges are sorted from maximum to minimum and converted from ft^3/s to m^3/s by dividing by 35.31. The frequency is determined from this ranking by dividing the rank position by the record length (in this case 53 years). Log of the frequency and Log of the discharge are taken using the simple transform utilities. Log Discharge is plotted as a function of Log

Frequency. The slope of the regression line is determined manually (to have the flexibility to ignore the data tail), and the values of β, C and C' are calculated as indicated. Note than the drainage area above the gauge of 792 mi^2 is given in the table caption. Plots should be made of Log Discharge versus Log Frequency and Discharge versus Log Frequency. A copy of the Excel data file should be labeled with river, location, and page number reference to the data table and saved on the class diskette.

The method is most easily understood by just selecting a few data points from the entire list. The most critical data are the larger discharge events. The data I selected are shown by checks on figures that will be supplied with the lab. These figures illustrate how data are plotted and the parameters determined.

Questions to Consider:

1. Is the fractal fit for smaller drainage basins better?
2. Are the β factors for small drainage basins more erratic than those for larger basins?
3. Is C' smaller for large basins than for small basins?
4. Why might floods be fractal (scale invariant)? Why might larger drainage basins be less fractal than smaller drainage basins, and the traditional engineering approach better for large drainage basins and the fractal description better for small?
5. Is there a regular trend of b and C' as you go up river?

Chapter 15: Geophysical Techniques

Drilling is the only sure way to know for sure what is in the subsurface. However drilling recovers only a very small sample and is expensive. For this reason, indirect techniques can be very important. They allow the subsurface to be probed without drilling and allow interpolation of data between drill holes.

The indirect geophysical techniques that are useful in probing the sub-surface involve measuring and interpreting the gravity field, and sending sonic or electromagnetic signals into the subsurface and recording and interpreting their reflections.

Examples of the use of these indirect techniques will be given in class.

IV. CHEMICAL CYCLES AND CLIMATE CHANGE

Chapter 16: What We Know About Changes in Climate

The following are some observations bearing on the earth's temperature history:

(1) The sun's luminosity has increased ~20% over the last 3.8 billion years. If anything, it appears that over this period the earth's average temperature has decreased.

(2) World temperatures decreased by ~10°C at the latitude of England from the Eocene (~40 Ma) to the end of the Pliocene (1.8 Ma). Alligators and broad leaf semi-tropical plants in the Eocene clays in England were replaced by northern arboreal forests.

(3) The ice ages started in the Pleistocene, 2 million years ago. The gradual waxing and sudden waning of the ice sheets in Canada and the Scandinavian countries correlates with the 90,000 and 120,000 periods of variation of the eccentricity of the earth's orbit around the sun (one of the so-called Milankovitch cycles). These cycles are predictable by celestial mechanics and are due to the influence of the other planets on the earth's orbit, tilt, and the orientation of the tilt. There have been 15 to 20 ice ages over the last 1.8 Ma.

(4) Pollen indicated that the temperature increased abruptly at the close of the last ice age about 13,5000 years ago. The changes in temperature were global. Glaciers in the southern and northern hemisphere and at both high and low latitudes retreated at the same time. The glaciers retreated about 1 km in elevation worldwide.

(5) There was a maximum in temperature about 1100 AD when Greenland and Iceland were settled.

(6) There was a little ice age which lasted from ~1500 to 1860. The Danube froze in 1860. Skating was common on the canals in Holland. Routine winter travel from Virginia to Washington DC was by sleigh.

(7) From 1860 to present the temperature of the earth has been warming. All mountain glaciers have retreated by about 200 m in elevation since 1860, suggesting a warming of ~0.6°C.

(8) Over roughly the same period (1860 to present) humans have caused about 300 billion metric tons (BMT) of carbon to be introduced into the atmosphere. About half has remained in the atmosphere, causing an increase in atmospheric carbon content of 140 BMT. The other 160 BMT has presumably been dissolved in the oceans or incorporated into forest growth in the northern hemisphere. Man's addition of Carbon has been about half from deforestation (which causes plant carbon which would normally

be buried to be oxidized to CO_2) and half from the burning of fossil fuels. Table 1 provides a summary. See also Figure 16.1

(9) High resolution temperature data has been recovered from ice cores in Greenland and Antarctica. This data shows that temperature can change extremely rapidly (within a few years) to various degrees of "ice house" or "green house" conditions. These rapid temperature changes are known as the "flickering switch".

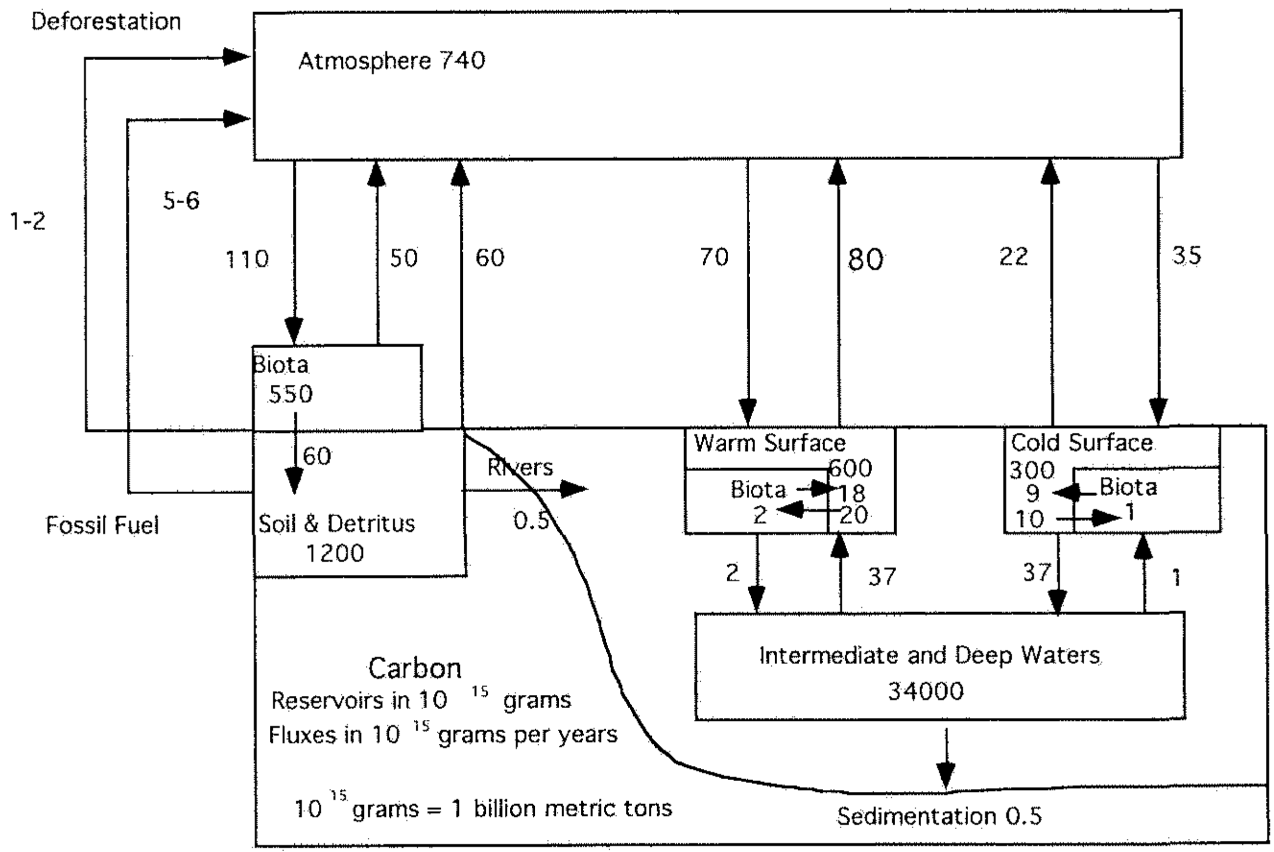


Figure 16.1 The global carbon cycle. Current estimates of the major reservoirs (in 10^{15} grams of carbon) and fluxes in 10^{15} grams of carbon per year) involved in the global carbon cycle. Modified slightly from Oceanus, v. 29 #4, 1987, p. 11.

Table 1: **Carbon in the Atmosphere** (Oceanus, 1986/6, v. 29 #4). BMT stands for billion metric tones.

Changes in Atmospheric Carbon			
	1980	346 ppmv CO_2	740 BMT C
	1860	280 ppmv	600 BMT
	Change	66 ppmv	140 BMT
Additions of C to the Atmosphere			
	Deforestation		150 BMT
	Burning Fossil Fuels		150 BMT
	Total		300 BMT
Present Rate of Addition of C to the Atmosphere			
	Deforestation		~1 BMT/yr
	Burning Fossil Fuels		5 BMT/yr
	Total		6 BMT/yr
	Atmospheric C increase		2.5 BMT/yr
	Ocean C increase		2.5 BMT/yr

The **current rate** of deforestation is ~70,000 to 100,000 km^2 per year or an area approximately the size of Mass, New Hampshire, and Vermont combined. **Predictions** indicate an approximate doubling of atmospheric CO_2 to ~600 ppmv by the end of the next century. **The Concern** is that the rapid anthropogenic rise in CO_2 will affect global climate. **The Question** is what can science say about this possibility.

Chapter 17: Long Term Climate Control

Let us try to analyze these changes with some simple calculations. First consider the effect of increasing the sun's luminosity by 20% on the black body temperature of the earth.

- The solar flux is $1.34 kW/m^2$.
- The earth's albedo is 0.3
- Therefore the earth intercepts $0.7 \times 1.34 kW/m^2$ from the sun.
- If the earth radiates this energy as a black body from its full surface area of $4\pi r^2$, the black body temperature of the earth will be:

  $0.7 \times 1.34 kW/m^2 \times (1/4) = \sigma T^4$,

  where σ is the Stephan-Boltzman constant with a value of 5.6×10^{-8} $W/m^2\text{-}K^4$.

- Solving for T gives T[°C]= -19.4°C.

- If the solar flux were 80% of the present value some time in the past, it is easy to show by repeating the above calculation that the black body temperature would have been 13.7°C cooler.

- Therefore each 1% increase in solar flux, all things being equal, will increase the temperature of the earth by about 0.7°C. Note that the figure you get depends somewhat on the temperature range calculated. Verify this by calculating the change in black body temperature for 0% and 20% earth albedo.

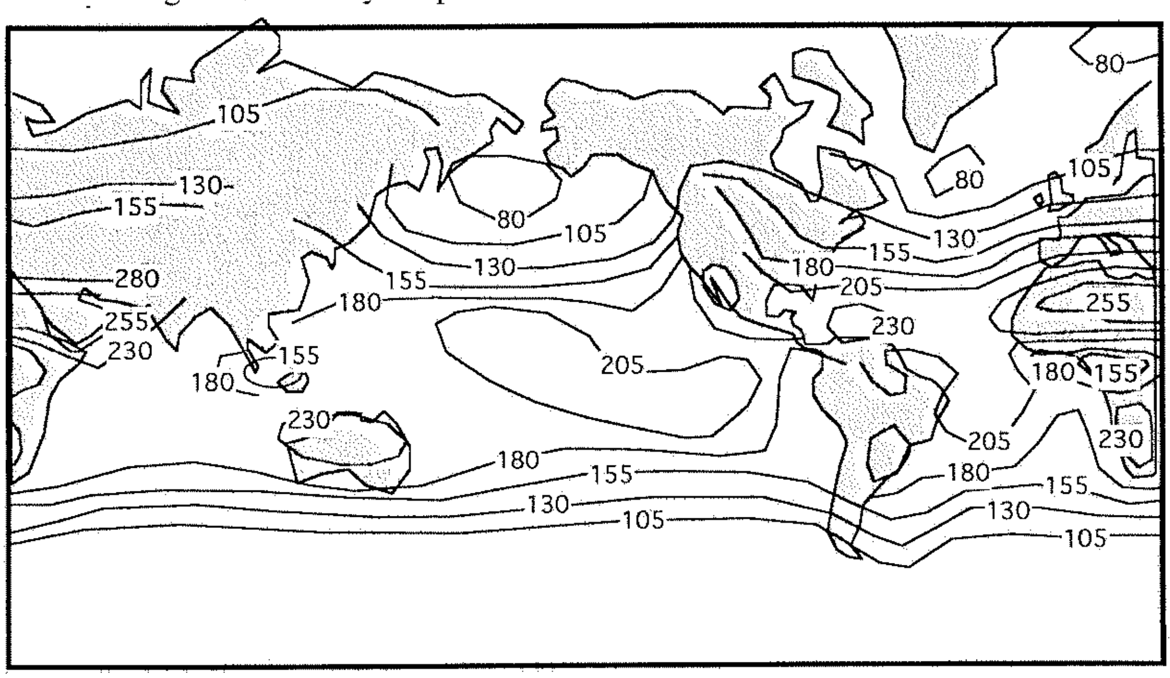


Figure 17.1. Solar flux absorbed by surface in watts/m2. Simplified from Bearman, Editor, Ocean Circulation, Pergamon Press, New York, 1989.

From Figure 9.1 you can see that about 51% of the solar radiation flux actually is absorbed by the earth's surface. The albedo of 0.3 indicates that 30% is reflected and this figure shows that and additional 19% is absorbed by water vapor, dust, O_3, CO_2, and clouds. The flux at the equator is thus on average 217 W/m^2. Note that radiation is absorbed at the equator along a band 2r long, but radiated from a circumference $2\pi r$ long. Figure 17.1 shows that the actual average absorption of solar radiation varies from 155 to 280 W/m^2. What is the reason for this range of ±30%? What does this suggest about the earth's temperature and clouds, dust, etc.? How much would be earth's temperature be lowered if the incident solar energy were 155 W/m^2 rather than 217 W/m^2, all other factors remaining equal? (Answer, the temperature would be lowered about 20°C because there has been a 28.5% decrease in incident energy and the black body temperature of the earth decreased by -0.7 °C per percent decrease in solar energy absorbed by the earth's surface). All other factors, such as the greenhouse effect (which is increased by water vapor) atmospheric circulation, ocean currents, etc., are certainly not constant, but the calculation does clearly show the importance of albedo on the average temperature of the equatorial regions.

The fact that the average temperature of the earth has generally been very stable suggests the temperature is buffered. A **long term** buffer is provided by the coupling of the weathering cycle to the rock cycle as discussed by Broecker in Chapter 7, "Making it Comfortable", in *How to Build a Habitable Planet*. Broeker's argument is: Rocks are chemically dissolved by carbonic acid (H_2CO_3) which is just carbonated water (CO_2 dissolved in water, e.g., $CO_2 + H_2O = H_2CO_3$). In the oceans, calcite is used by marine organisms to build their skeletons. When they die their skeletons (calcium carbonate) and some or all of their flesh or plant fiber (organic carbon) is incorporated as sediment on the sea floor. Some CO_2 is released in the process. When this material is subducted it is heated to temperatures high enough to convert the carbonates to silicates and release more CO_2. Over long periods, the fluxes (positive and negative) of carbon to the atmosphere are balanced and the CO_2 content of the atmosphere stable. The reactions are summarized below:

Weathering Reactions:

$CO_2 + H_2O + CaCO_3 = 2HCO_3{-} + Ca^{++}$
$2\ CO_2 + CaSiO_3 + 3H_2O = 2HCO_3{-} + Ca^{++} + H_4SiO_4$

Ocean Carbonate Precipitation

$2Ca^{++} + 4HCO_3{-} = 2CaCo_3 + 2CO_2 + 2H_2O$

Subduction Zone Metamorphism

$CaCO_3 + SiO_2 = CO_2 + CaSiO_3.$

Notice that the reactions are balanced. Three units of CO_3 are removed from the atmosphere by weathering and three returned to the atmosphere by reactions in the oceans and at subduction zones. The weathering of dolomites and the conversion of Mg^{++} to Ca^{++} at the mid-ocean ridges has been omitted for simplification.

Weathering reactions, like almost all chemical reactions, proceed faster at higher temperatures. If CO_2 controls temperature, with this clue, can you explain how the plate tectonic rock cycle acts as a buffer to control temperature? For example, if the temperature were to fall for some reason, what processes would tend to increase it again? If temperature were to rise what processes would decrease it? In other words, show how the hydrologic (which carries CO_2-saturated rainwater over the mountains and precipitates it) and the rock cycle interact in a fashion that will tend to stabilize the earth's temperature. What other buffer mechanisms can you imagine? How about the growth rate of vegetation which is stimulated by both higher T and higher CO_2 levels?

We thus have at least some possible explanations for the general stability of the earth's temperature over the long term. But stability is cannot be the whole story because long therm changes in climate have occurred. Specifically what is the explanation for the decrease in temperature that occurred between the Eocene and Pliocene which cooled the earth to the point that we have experienced 15 to 20 glacial cycles?

What was happening at the time? From 40 to 2 million years ago two major events occurred: First India collided with Asia causing the uplift of the 4.5 km high Tibetan Plateau. This plateau has a major effect on the global atmospheric circulation, accentuates the monsoons, etc. Second North America overrode the Yellowstone and Ratan plumes, causing the uplift of the western U.S. Two million years ago coinciding exactly with the onset of continental glacial cycles in the northern hemisphere, the Isthmus of Panama uplifted, separating the Atlantic and Pacific Oceans. Numerical calculations show that the rise in the north American and Asian plateaus would have had a significant cooling effect on the northern hemisphere. The cooling is not as large as is required for a full explanation of the Pleistocene ice age, but the calculations do not consider the effects of ice on albedo (clearly a major effect), or changes in albedo due to changes in cloud cover. These deficiencies are common to almost all present climate computer models. All considered, plateau uplift could have caused the gradual Eocene to Pliocene cooling.

What about the abrupt changes in temperature associated with the ice age glacial cycles? While we are at it, what causes the equally abrupt smaller changes in temperature such as those that occurred at the end of the little ice age? Good candidates for these changes are ocean or atmospheric circulation - in particular the thermo-haline circulation that has its source in the Atlantic and which was aided by the rise of the Isthmus of Panama.

Chapter 18: Ocean Circulation and Temperature Control

The effects of the Gulf Stream on the temperature of the North Atlantic and surrounding land masses is well known. What may be less appreciated are the effects of the thermo-haline circulation. The wind-driven and thermo-haline-driven circulations operate independently of one another, and both are important in transporting heat.

Consider thermo-haline circulation. Water is cooled in the northern and southern extremities of the Atlantic. In addition the water is made even more dense by virtue of its relatively high salinity. Part of the reason for the higher salinity of Atlantic water is the transport of water vapor across the Isthmus of Panama. Figure 18.1 shows this effect. Part of the reason is the formation of sea ice (which rejects salt and so makes the residual sea water saltier). The formation of sea ice can be episodic, as when winds open large areas in the middle of the Antarctic ice sheets called polynyas. The cold (~2°C) salty water circulates south from the Arctic (under the Gulf Stream) and north from the Antarctic to the latitude of northern Argentina, and then turns east and flows into the Indian Ocean, and ultimately into the Pacific.

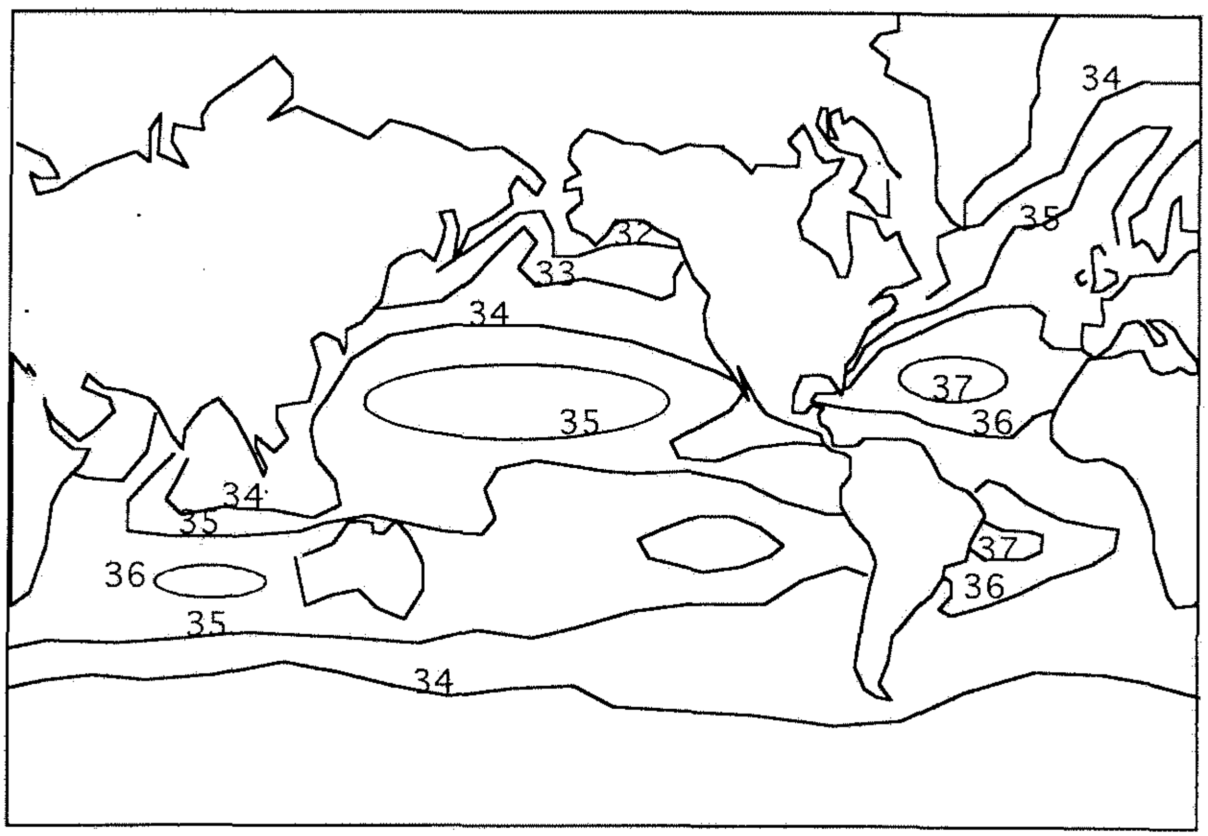


Figure 18.1. Mean annual surface salinity of the oceans in parts per thousand total dissolved solids. Modified from Bearman, Editor, Ocean Circulation, Pergamon Press, New York, 1989.

The magnitude of the thermo haline circulation is large and its effects on temperature in the North Atlantic major. The flow can be estimated by conservation of salt. The flux of water vapor across the Isthmus of Panama that produces the salinity anomaly in the shallow waters of the Caribbean shown in Figure 18.1 is $0.35 \pm .1 \times 10^6$ m^3 of evaporated seawater per second. Since seawater has a salinity of 35 parts per thousand, this means that about 1.2×10^9 grams of salt are added to the Atlantic each year. If the salinity of the Atlantic is not increasing 1.2×10^9 grams of salt must leave the Atlantic per year. The water that exits the Atlantic in the deep haline circulation current shown in Figure 9.8 carries out the salt carried in by the Gulf Stream (Figure 9.7) The salty, warm Caribbean water is cooled and sinks in the Norwegian, Greenland and Labrador seas, and is replaced by Arctic surface water with a salinity 0.6 per mil less salty. It is augmented by salty water formed where pack ice is produced in the Arctic Sea.

Salt balance of just the salt flux across the Isthmus requires that the thermohaline flow be 20×10^6 m^3/sec. This flow is arrived at by balancing the inflow and outflow of salt from the North Atlantic.

Salt source in the Carribbean = $C_{seawater}*E_{Carribbean}$=(35ppt)($0.35 \times 10^6 m^3/s$).
(C denotes concentration and E denotes evaporation)

Balance:
Flow of extra salt into the North Atlantic= salt source in the Caribbean
or $C_{seawater}*E_{Carribbean}=\Delta C_{N.Atl} \cdot F_{N.Atl}$.

Net export of salt from the North Atlantic=$\Delta C_{N.Atl} \cdot F_{N.Atl}$=(0.6 ppt)$F_{N.Atl.}$

Thus the haline flow of the North Atlantic is:
$F_{N.Atl}$=(35ppm)($0.35 \times 10^6 m^3/s$)/0.6ppm=$20 \times 10^6 m^3/s$.

At steady state (a condition where the salinity of the north Atlantic area is not changing) the flux of salt into the north Atlantic region must equal the flux of salt out. The flux in is assumed here to equal the extra salt carried north by the Gulf Stream that was produced by the net transport of water vapor, $E_{Carribbean}$, across the Isthmus of Panama ($0.35 \times 10^6 m^3/s$). The export of salt from the north Atlantic is estimated by the difference in salinity between the Arctic water drawn into the haline circulation, and the salinity of that circulation when it leaves the Arctic. This difference is estimated at 0.6 ppt. From the above calculation we conclude that every second 20 million cubic meters of water enter and leave the North Atlantic. This is the equivalent flow of 1180 Mississippi Rivers (see p 73).

The water flowing into the North Atlantic has a temperature of ~10°C, and the water leaving a temperature of ~2°C. Thus each m^3 of water carries in 3.34×10^7 joules. The total heat transport to the North Atlantic by the thermo-haline circulation is thus 6.7

x 10^{14} J/year. The area of the North Atlantic Region is ~4750 x6000 km^2 so this heat transport is equivalent to 33 W/m^2 or 10% of the total average incoming solar energy of 1340/4 W/m^2. The thermo-haline driven circulation of the Atlantic is equivalent to a 10% change in albedo! You should be shocked at the large size of this heat flux. Why?

For comparison, the greenhouse warming due to a doubling of the CO_2 content of the atmosphere from 300 to 600 ppm is estimated to be ~5°C in the north Atlantic area. This is equivalent to $\frac{5°[C]}{0.7°[C/\%]}$ or an ~7% increase in the solar flux. Clearly thermo-haline circulation is competitive with this eventual effect of anthropogenic CO_2. The question of what controls the earth's short term (human scale) temperature changes remains open: Ocean currents, CO_2, Milankovich cycles, changes in the solar constant, changes in cloudiness (tied to changes in the intensity of cosmic radiation), or something else are all possible. Ice volume correlates with atmospheric dust, Milankovich Cycles, Atmospheric CO_2, atmospheric δD, atmospheric $\delta^{18}O$, etc.), but correlation does not indicate cause. Except for the Milankovich cycles, all these correlations could be consequences of temperature change or glaciation rather than its cause.

Chapter 19: Mineral Resources

Mineral resources are "the endowment of useful minerals ultimately available commercially" **Ore** is material that can be mined for a profit under today's economic conditions. (The Earth, p.566).

Most ore deposits require a source, a transport agent, and selective deposition of valuable materials. Transport can be achieved by any fluid (water, magma, atmosphere). Deposition can be triggered by changes in pressure or temperature, or chemical change. The processes that make ore deposits need not be dramatic- it can be business as usual. An analogy is kids with little red wagons. If neighborhood kids are organized to pick up rocks and deposit them in a particular yard, quite a pile of rocks can result. At any instant nothing unusual would appear to be taking place. But the pile of rocks in on yard could appear in retrospect to be quite remarkable.

Table 19.1 lists a number of transport properties and the ore deposits that are produced. Magmas form ore deposits when chemical additions such as organic (reducing) material cause Ni and Co to precipitate (as in the Norilsk deposits that lie in the feeder pipes for the Siberian flood basalt flows). The outgassing of aqueous volatiles from magmas precipitate Cu, Mo, and Au when the volatiles decompress (boil). This produced porphyry deposits.

Sedimentary deposits are formed when valuable material chemically precipitates from the ocean or lakes. Evaporites such as salt may accumulate to kilometer thickness in restricted basins. Iron was leached from the land surface and precipitated in the world's oceans when oxygen levels rose in the atmosphere 2.5 Ga. Our main supply of iron ore comes from banded iron formations that formed in this way and at this time.

Metamorphic deposits form when compaction squeezes brines out of basins as sediments accumulate. Mississippi Valley Type lead zinc deposits form when these metals precipitate from the brines when they cool as they exit the basin. Organic material is slowly cooked as sediments are buried and warmed. The mobile hydrocarbon products (oil and gas) of this maturation migrate upward (because oil and gas are lighter than the water that fills the sediment pores) and are trapped against impermeable strata.

Weathering can produce mineral deposits. Aluminum is very insoluble and remains after everything else is leached from a rock or soil. Bauxite deposits are the result. Porcelain clay is a clay purified by an unusual amount of water washing. In some cases the drive for the washing appears to have been radiogenic heat production in associated radiogenic granites.

Ore deposits are important in their own right, because we need metals, salt, phosphates, etc., and because of what the deposits reveal about geologic processes and events that have affected the earth. They are particularly valuable sources of information because their geology has been interrogated and compiled by drilling, mining, and geophysical studies. Millions have been spent collecting information valuable for much more than just the definition and mining of the ore itself.

Process	Deposit Formed	Mineral Resource
Igneous Process	Magmatic Segregation	Chromium, Nickel, Copper
	Pegmatites	Lithium, Tantalum
	Hydrothermal Deposits	Copper, Lead, Zinc, Gold, Tin
Sedimentary		
Clastic Rocks	Stream Deposits	Sand, Gravel
	Placer Deposits	Gold Platinum,Diamonds, Tin
	Dune Deposits	Sand
	Loess Deposits	Soil
Chemical Precipitates	Evaporite Deposits	Halite, Gypsum, Borax, Trona
	Marine Sediments	Banded Iron Formation
Organic Precipitates	Hydrocarbon Deposits	Oil, Natural Gas, Coal
	Marine Deposits	Limestone
Metamorphic Process	Contact Metamorphism	Tungsten, Copper, Tin, Gold
	Regional Metamorphism	Gold, Talc, Asbestos
Weathering & Groundwater	Residual Weathering Deposits	Nickel, Iron, Aluminum, Gold
	Groundwater Deposits	Uranium, Sulfur, Travertine
	Geothermal Wells	Hot Water, Electricity

Table 19.1. Major Geologic Processes that Form Mineral Resources

Chapter 20: Energy Resources

20.1 World Consumption of Oil and Gas

Perhaps the biggest barrier to understanding is unit translation. Here are some useful conversions for the yearly worldwide hydrocarbon consumption.

F=ma, where the units are [kg-m/ sec^2]=1 Newton

Work = $\int F \bullet dx$, with units of: Newton-m = joule

1 barrels of oil (bbl) =6120 *10^6 Joules = 6.2 Gigajoules
=5.8*10^6 BTU's
=6,000 ft^3 Gas

World Consumption (WC) of Oil and Gas:
=25*10^9 bbl/yr
=1.36*10^6 kW-hours
=1 WC
=1.57*10^{20} joules/yr,
or an average rate of 5 TW of consumption (~20% of Earth's surface heat flux)

20.2 The Sun and Magma as Energy Sources

Hydrocarbons are fossil solar energy. Here is how solar input (and heat from the earth-geothermal energy) translate to WC units:

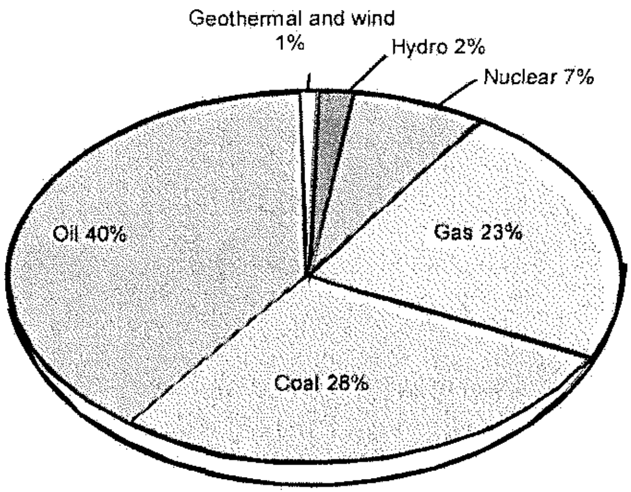
Solar Input: 1.34kW/m^2 * πr_E^2
=5.35*10^{24} J/yr
=21,200 WC intercepts the Earth annually

Total Heat From the Earth: 70mW/m^2*(4* πr_E^2)
=10^{21} J/yr
=6.4 WC

Magmatic Heat: 17 km^2 new crust formed/yr, 6.5km thick
=110 km^3/yr
=110*10^9 m^3*2700kg/m^3*1000J/kg-°C*1200°C
=2.9*10^{20} J/yr
=1.9 WC

20.3 Current Energy Sources and Per Capita Consumption

Petroleum, coal and natural gas, known as fossil fuels, provide 90 percent of all commercial energy used worldwide (Figure 20.1). Renewable energy sources (geothermal, wind, hydrothermal) produce about 3 percent, and nuclear power provides 7.



Distribution of the world's energy needs.

Figure 20.1 Worldwide commercial energy production
(Adapted from Env.Sci:A Global Concern, p.465, 1997)

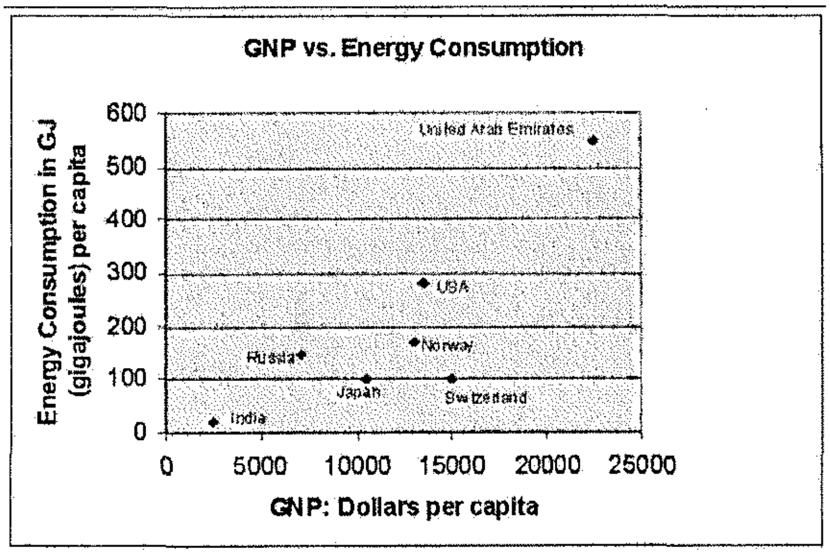
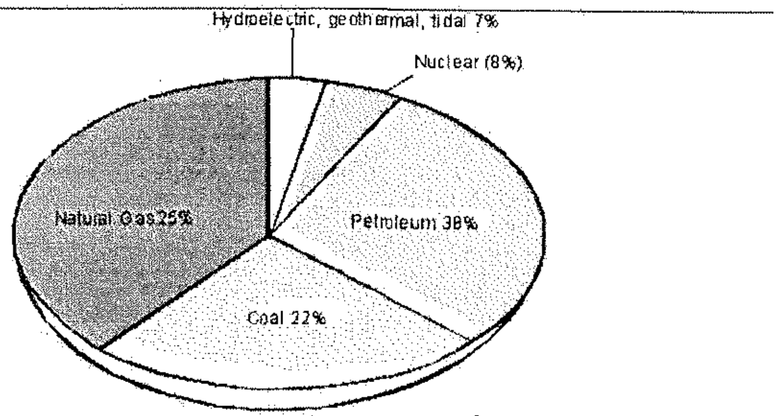
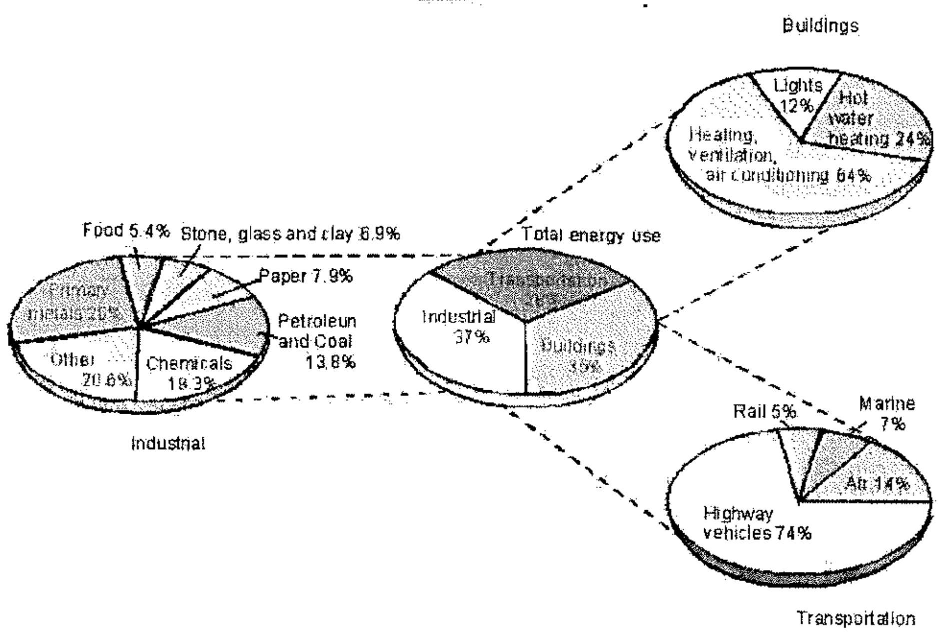


Figure 20.2 Per Capita Energy Consumption and GNP
(Environmental Science: A Global Concern, p.465, 1997)

India, which has a relatively low standard of living compared to Western countries, consumes about 25 gigajoules (GJ) per capita per year (~4 barrels of oil equivalent), while in the U.S. each person consumes about 300 GJ per year, which is equivalent to 50 barrels of oil per year per person. Overall ~1/5 of the world's population consumes ~2/3 of the commercial energy supply. Energy consumption correlates directly to standard of living. Thus as poorer areas raise their living standards, and as the world population grows, more energy will be needed.

20.4 Energy Flow for the United States





Figures 20.3 & 20.4 Energy Consumed in the United States. Energy Flow diagram for the US in 1983.

(Adapted from Earth's Dynamic Systems, p.652, 1998)

In the United States, about 85 percent of the energy used each year is provided by non-renewable fossil fuels (Figure 20.3). 37 percent of the energy used in the U.S. is consumed by industry (Figure 20.4) and over ¼ of this energy is used to mine, smelt and mill primary metals. The chemical industry uses about 20 percent of industrial energy share to manufacture organic chemicals, fertilizers and plastics. The manufacturing of food, stone, glass, and paper also require large amounts of energy, about 20 percent of the industrial share.

Commercial and residential buildings consume 35 percent of the U.S's energy, with most of this energy going towards heating, ventilation, and air conditioning (64 percent). The remaining energy is used to heat water and to provide lighting.

Transportation, including cars, airplanes, trains and ships, consumes 28 percent of all energy used each year. 98 percent of this energy is provided by liquid fuels (petroleum), while the remaining 2 percent is provided by electricity and natural gas. Highway vehicles use a tremendous amount of energy, nearly ¾ of the transportation energy share.

20.5 Non-Renewable Resources-Fossil Fuels

Over the past 20 years, estimates of world wide fossil fuel reserves have increased significantly. According to the World Resources Institute, proven recoverable reserves of natural gas has risen 140 percent, while the amount of petroleum reserves rose 60 percent between 1973 and 1993 (Figure 20.5).

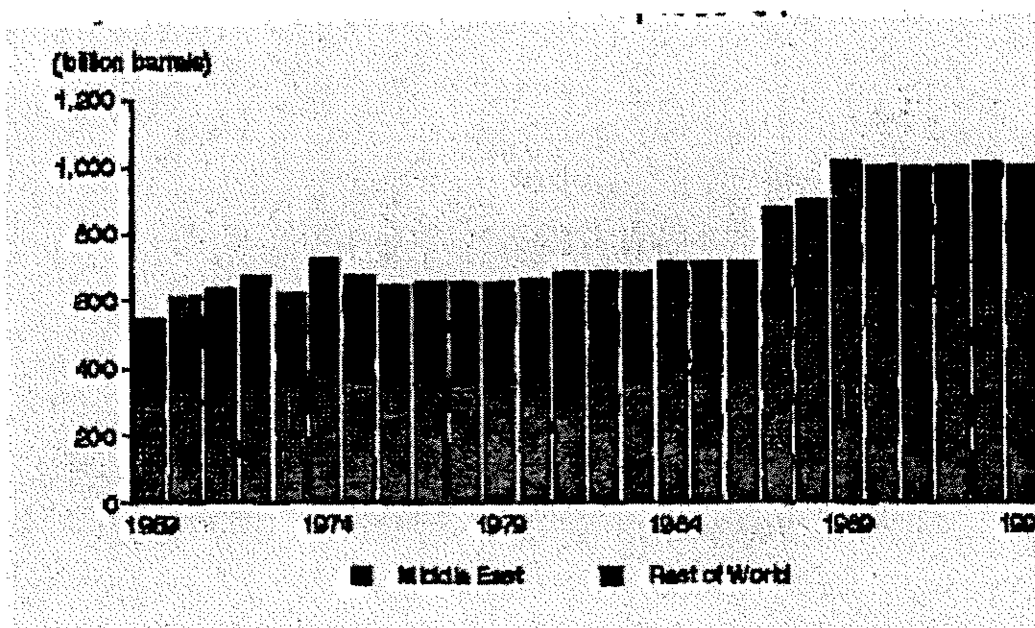


Figure 20.5 Proven Petroleum Reserves, 1969-1994.
(World Resources Institute, www.wri.org)

Petroleum

Petroleum and natural gas are made up of hydrocarbons, which are found in plants, animals, algae, and microscopic organisms. When these organisms died millions of years ago, they accumulated on the sea floor and did not decay completely due to their quick burial in the marine sediments. High pressures and temperatures concentrated these organic molecules into the high-energy compounds that are used today for fuel. Oil and natural gas will form only if some basic conditions are met: 1) The environment must be oxygen poor in order to prevent decay of organisms, 2) Organic material must be buried deep for sufficient heat and pressure to concentrate material, 3) Reservoir rock (such as sandstone) must be present in order to allow petroleum migration, and 4) Cap, or barrier rock, must be present to trap fluids in a confined space.

Petroleum is found all over the world (Figure 20.6) with the largest supply found in Saudi Arabia (almost 1/5 of the worlds reserves). Petroleum provides 40 percent of the world's commercial energy, which is currently 25 billion barrels a year. Many media sources and textbooks predict that the oil reserves will be depleted in 40-50 years. This is not true. There is an abundance of oil in both tar sand deposits (which are currently economically viable) and oil shale deposits (which will be usable as long as we can get more energy from them than it takes to extract the oil from them). At the present rate of consumption, the tar sand deposits contain 150-250 World Consumptions (WC), while the oil shale deposits contain 13,720 WC. A problem that will exist in the U.S in the coming years deals with import policy. As oil consumption increase and total U.S. oil production decreases, more oil will have to be imported from abroad. This now becomes a political issue, as the U.S. government has to ensure that oil-rich countries will be willing to sell oil to the U.S. in future years, and must find a way to pay for this oil.

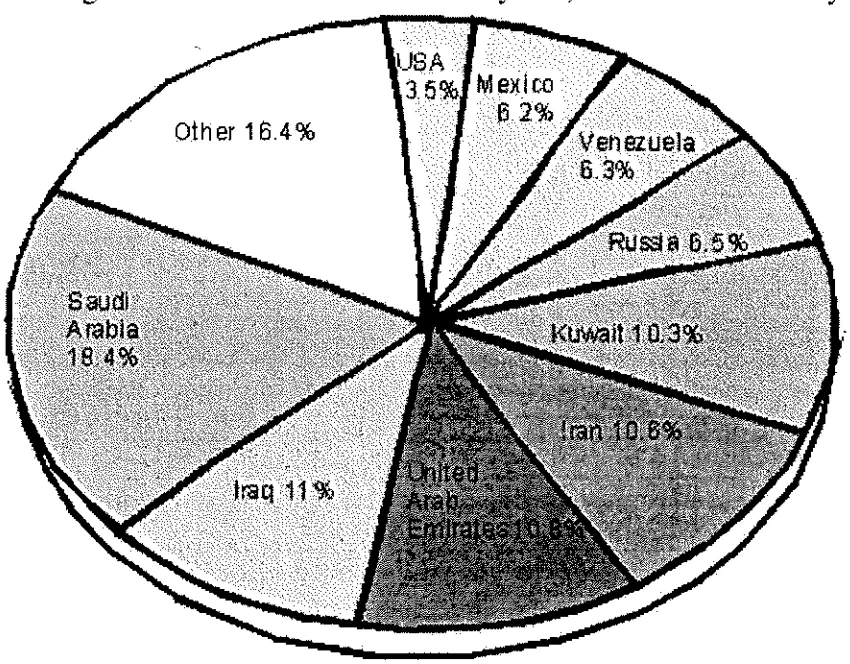


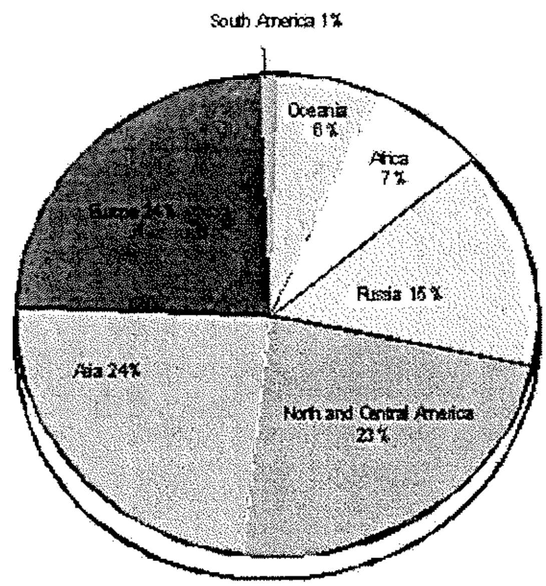
Figure 20.6 WorldWide Proven Oil Reserves
(Environmental Sciences: A Global Concern, p.470, 1997)

Coal

Coal, like petroleum, is made from fossilized organic matter that has been buried and preserved in sediments. Most of these deposits were formed between 285 and 360 million years ago, during the Carboniferous period. Coal can be found in vast slabs 100 meters thick that sweep across an area tens of thousands of km^2. If it were possible to mine all of this coal and burn it for energy, we would have a several thousand-year supply, assuming we use the coal at present day consumption levels. But, because much of this cannot be mined due to its depth and location, present rates of consumption suggest that our coal supply will last about 200 more years.

Coal and other solid fuels (lignite, peat) supply approximately 28 percent of the worlds commercial energy, and is found in large quantities in Asia, Europe and the Americas. Despite the fact that coal is relatively abundant as an energy source, there are many drawbacks to mining and burning coal. Burning coal releases many toxic metals-lead, mercury, zinc-that were absorbed by plants and concentrated during coal formation. Since coal is 10 percent sulfur by weight, if sulfur is not removed from the coal before it is burned, SO_2 and SO_4 are released into the atmosphere. Every year in the United States, burning coal releases 18 million metric tons of SO_2, 5 million metric tons of nitrogen oxides, 600,00 metric tons of carbon monoxide and a trillion metric tons of carbon dioxide.

Figure 20.7 Coal Resources. (Environmental Sciences: A Global Concern, p.470, 1997)



Natural Gas

Like petroleum, natural gas is made up of hydrocarbons and is usually in the form of methane (CH_4). Natural gas is currently the fossil fuel with the largest growth in consumption, rising more than 70 percent from 1973-1993. Natural gas is a cheap, relatively clean burning fuel, releasing half as much CO_2 as coal when burned. Russia and Asia have the largest percent of proven reserves (Figure 20.8). Currently, there is 73,000 metric tons of natural gas available and the proven reserves contain a 60-70 year supply of natural gas.

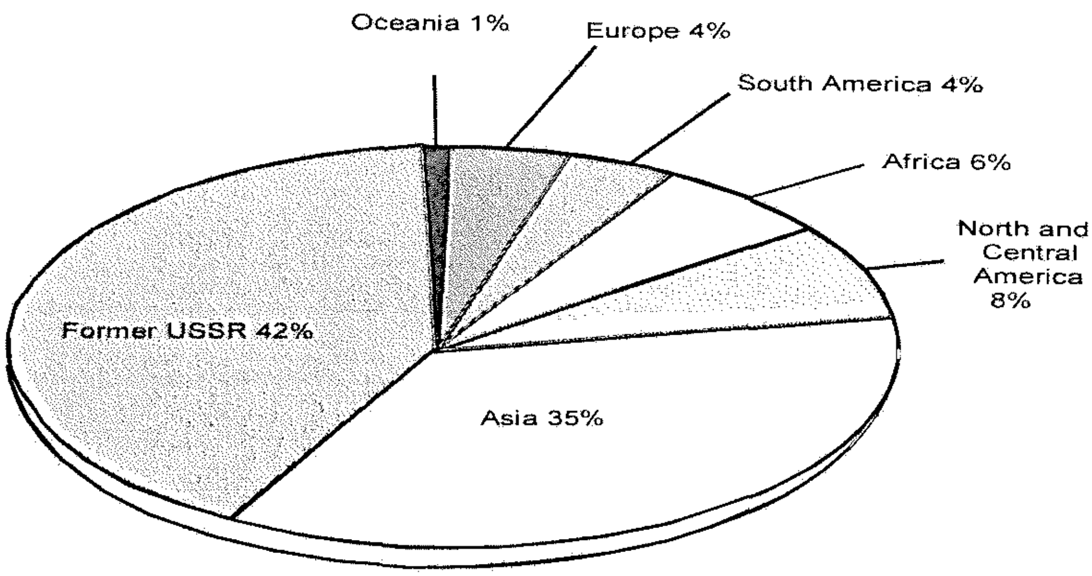


Figure 20.8 Proven –in-place natural gas reserves
(Adapted from Environmental Science:A Global Concern, p.473, 1997)

Appendix

Achondrites – Stony meteorites that lack chondrules.
Adiabatic – A process fast enough or in a system sufficiently insulated that no heat escapes or enters the system.
Albedo – The fraction of the total radiation that is reflected from a planet and therefore does not interact with it (warm it, etc.).
Asthenosphere – Subdivision of the mantle, located below the lithosphere. Typically extends from ~100 to ~250 km depth.
Basalt - An igneous rock consisting mostly pyroxene and plagioclase, with some olivine. The dominant rock of the ocean crust (mid-ocean ridge basalts) and ocean islands (ocean island basalts)
Batholith – Large (>40 mi sq.) mass of igneous rock formed when magma is emplaced at depth and slowly cooled.
Black body - an ideal body or surface that completely absorbs all radiant energy falling upon it with no reflection, and that radiates at all frequencies with a spectral energy distribution dependent on its absolute temperature.
Carbonaceous Chondrites – Type of chondrites containing carbon. The mineral phases in some of these meteorites are destroyed by moderate heating.
Calcite – A mineral consisting of calcium, carbon, and oxygen. Abundant in limestone and marbles.
Chondrites – Stony meteorites containing millimeter size spheres or chondrules.
Coefficient of thermal expansion (α) – Fractional increase in volume per unit change in temperature.
Conglomerate – Sedimentary rock, composed of rounded, water worn pebbles cemented together.
Crust- the upper part of the lithosphere that is chemically different (differentiated) from the mantle.
Felsic rock - Light colored rock composed primarily of feldspar (crystalline aluminum silicates), feldspathoids and quartz.
Granite - An igneous rock consisting mostly of quartz and feldspar.
Half Life – The time required for half of the radionuclides of a given type to undergo radioactive decay.
Heat capacity (*c*) – Amount of heat needed to raise a material's temperature 1 degree.
Homogenizing event – An even that resets the radio clocks of the radioactive materials. (i.e melting).
Hydraulic conductivity (*K*) – A measure of a porous materials ability to transmit water. R the flux of water to the gradient in hydraulic head. Units: [m/s]
Hydraulic head- The standing height of water in a lined well, perforated at the point of measurement.
Igneous rock – Formed by crystallization of molten magma.
Inner Planets –Mars, Earth, Venus, Mercury.
Injection well/recharge – A well where water are flowing into the ground, hence recharging the subsurface.

Intrinsic permeability (k)- Measure of the ability of a material to transmit fluid under a pressure gradient. Dimension is m^2. Relationship with hydraulic conductivity: $K = \kappa\left(\frac{\rho g}{\eta}\right)$, where ($\rho$ = density, g = acceleration, η = viscosity)
Island arc – Island chain formed where the oceanic lithosphere is subducted into the mantle.
Isostasy – Floating equilibrium. An object must displace it's own weight in order to float. The Earth's lithosphere floats on the asthenospheric "fluid".
Isotope – Nuclides with the same number of protons but a different number of neutrons are isotopic brothers.
Laramide time - A period ~ 60 Ma when the Rockies were uplifted.
Latent Heat – Heat that input to a system that is required to cause phase transformation without increasing the system's temperature.
Limestone – A sedimentary rock consisting mostly of calcite.
Lithosphere – The rigid, cool outer layer of Earth.
Mafic rock – Igneous rock with low silica content and high iron-magnesium content.
Mantle – The 2900 km thick layer of Earth located underneath the crust.
Metamorphic rock – All rocks that have changed mineralogy as the result of heating in the subsurface to T>~200C.
Milankovitch Cycles – Changes in the Earth's orbit or the tilt of its axis. These changes change the amount of solar radiation striking different parts of the Earth at different times of year.
Non-gravitational water – Water that will not drain from a sediment under gravity.
Olivine $(Mg,Fe)_2SiO_4$ – A mineral consisting of iron, magnesium, silicon and oxygen. Abundant in basalts and mantle (ultramafic) rock.
Orogeny - The process of mountain formation especially by folding or thrusting of the earth's crust.
Outer planets – Saturn, Jupiter, Uranus, Neptune, Pluto.
Porosity - The ratio of the volume of void space in the soil to the total volume.
Peridotite – A rock consisting mainly of olivine (~60%), pyroxene, and an aluminous phase (either garnet at pressures greater than ~60km depths, spinel between 25-60km depths, or plagioclase between 0-25km depths). Believed to be the main mantle lithology.
Production well– Well where water is produced (caused to flow out of the well).
Protolith - The rock before it was chemically altered or mineralogically changed.
Pyroxene $(Mg,Fe)_2Si_2O_6$ and $(Mg,Fe)CaSi_2O_6$ – A mineral consisting of iron, magnesium, calcium, silicon and oxygen. Abundant in basalts and mantle rock.
Quartz SiO_2- A mineral consisting of silicon and oxygen. Abundant in granites and sandstones.
Rheology – The study of the deformation and flow of matter.
Rhyolite - A felsic igneous rock that are the extrusive equivalent of granite
River Discharge – The volume of water transported per unit time.
Sandstone – Course grained rock formed from lithified beach, dune or river sands.
Sedimentary rock – Formed from weathered products of preexisting rocks that have transported, depositioned and lithified.

Sialic - Relatively light rock that is rich in silica (Si) and aluminum (Al). Typically found in the continental crust.

Thermal conductivity (*K*) - A material property that describes the heat flows within a body that is caused by a gradient in temperature. Units: [cal/cm-sec-°C].

Thermal diffusivity (κ)- A material property that describes the rate at which heat diffuses through a body. It is a function of the body's thermal conductivity, density, and its specific heat.

Viscosity - A fluid's resistance to shear. Controls the flow of fluids.

Water table - The boundary between the zone where pores are filled with air and filled with water.

10 year flood – A flood with the highest river discharge magnitude compared to the rest of a data collection taken over a span of 10 years.

Glossary

$$\left[\frac{^{87}Sr}{^{86}Sr}\right] = \left[\frac{^{87}Sr}{^{86}Sr}\right]_o + \left[\frac{^{87}Rb}{^{86}Sr}\right]\left(e^{0.6931\, t/t_{1/2}} - 1\right)$$

$$j_o = -K\frac{\partial T}{\partial z}$$

$$F = -G\frac{Mm}{r^2}$$

$$z = 2\sqrt{\kappa t}$$

$$\Delta H_w = \frac{\rho_m}{\rho_m - \rho_w}\Delta H_a$$

$$r_c = \sigma T^4$$

$$g = G\frac{m}{r^2}$$

$$\Delta z = \alpha T_0\sqrt{\kappa t}$$

$$u = \frac{DF}{A}\left(\frac{t}{\eta} + \frac{1}{\mu}\right)$$

$$v_z = \frac{\rho g h}{2\eta k}(zk - 1)e^{kz}\cos kx$$

$$\Delta H_s = \frac{\rho_m}{\rho_m - \rho_s}\Delta H_a$$

$$u_z = \frac{\rho g h}{2\mu k}(zk - 1)e^{kz}\cos kx$$

$$\Delta H_c = \frac{\rho_m}{\rho_m - \rho_c}\Delta H_a$$

$$k = 1.7\left(\frac{1}{L^2} + \frac{1}{M^2}\right)^{0.5}$$

$$\Delta[o] = 10(\Delta t - 2)$$

$$R = \frac{\alpha\rho g d^3\Delta T}{\kappa\eta}$$

$$h = h_o e^{-\rho g t/2\eta k}$$

$$v = \frac{Q}{2\pi r b}$$

$$R_c = 1000$$

$$k = \frac{2\pi}{\lambda}$$

$$\tau = \frac{\phi}{Kk}$$

$$R = \frac{\alpha\rho g d^3\Delta T}{\kappa\eta}$$

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

$$1 + r + r^2 + r^3 + r^4 + r^5 \ldots = \frac{1}{1 - r}$$

$$u_e = \frac{\rho g h}{2\mu k}$$

$$V = \frac{K\Delta h}{L}$$

$$V = -K\nabla h = -\nabla \Phi$$

$$V_x = -K\frac{\partial h}{\partial x} = Kh_0 k e^{kz} \sin kx$$

$$V_z = -K\frac{\partial h}{\partial x} = -Kh_0 k e^{kz} \cos kx$$

$$h = h_o e^{-Kkt/\phi}$$

$$\Phi = -V[x\cos\alpha + y\sin\alpha] + \sum_{i=1}^{N} \frac{-Q_i}{2\pi b} \ln\left((x-x_i)^2 + (y-y_i)^2\right)^{0.5}$$

$$L = Q^{2.5}$$

$$K_z = \frac{d}{\sum_{i=1}^{n} \frac{d_i}{K_i}}$$

$$K_x = \frac{1}{d}\sum_{i=1}^{n} d_i K_i$$

$$T_o = \frac{4\pi b \phi x_o^2}{3Q}$$

$Q_i > 0$ for injection wells

Solar constant	**1340 W/m^2**
Density of upper mantle, crust, sediments, water	**3.17; 2.8;2.5;1.0 Mg/m^3**
Temperature of asthenosphere	**1200 °C (=1473K)**
Viscosity of deeper mantle	**10^{21} Pa-s**
Shear modulus of mantle	**7 x 10^{10} Pa**
Half life of ^{87}Rb	**49 Ga**
Stephan-Boltzman constant	**5.64 x 10^{-8} W/m^2-K^4**
Gravitational constant	**6.67 x 10^{-11} N-m^2/kg^2**
Radius of earth	**6371 km**
Diffusivity of mantle	**8x10^{-7} m^2/s**
Thermal conductivity of mantle	**3.14 W/m-K *or* 3.14 W/m-°C**
Heat capacity of mantle	**1170 J/kg-K**
Coefficient of Thermal Expansion	**3.33 x 10^{-5} K^{-1}**

Present-day fluxes.

Variable	Value
Plate spreading area (= consumption rate)	$2.7\,km^2/yr$
Mid-Ocean Ridge Basalt (MORB) flux	$4.9x10^{22}\,kg/Ga(=17\,km^3/yr)$
Ocean Island Basalt (OIB) production rate	$2.8x10^{21}\,kg/Ga$
OIB + Oceanic Plateau production rate	$7.6x10^{21}\,kg/Ga$
Subduction zone volcanism rate	$2.4x10^{22}\,kg/Ga$ (Schminke, 2000)
slab flux Φ_0 (=subduction mass flux)	$10^{24}\,kg/Ga$ (= 2.7km^2/a*100 km*3300kg/m^3)
mantle slab function $\Phi(z) = \Phi_0\left(1 - \frac{d \cdot z}{h_{mantle}}\right)$	$d = [0-1]$ (d=0 → all slab material reaches the core-mantle-boundary)
accumulated seafloor heat loss Q (function of seafloor age)	$3x10^{13}\left(\sqrt{age(Ma)}\right)J/m^2$ (Parsons and Sclater, 1977)

Present-day Mantle Energetics.

		$x10^{13}$ W	$x10^{-12}$ W/kg (of mantle)
Subducting slab: Thermal Energy	$Q_{slab} = \Phi_0 \Delta\overline{T} c_P$	-2.6	-6.27
Subducting slab: Potential Energy	$W_{slab} = \Phi_0 \Delta\overline{T}\alpha_0 g h_{mantle}\left[\frac{12-5d}{18}\right]$	1.2 [d=0.5]	3.0 [d=0.5]
Continental heat flux	56.6 mW/m^2	-1.1	
Mantle heat loss through continents	20 mW/m^2 (~175 km-thick continental tectosphere)	-0.4	-1
Net mantle heat loss	Sum of continental mantle fraction and subducting slab (=mean oceanic)	-3.0	-7.27
Earth's radioactive heat production (LOSIMAG)	U=21ppb, Th=80ppb,K=273 ppm (Zindler and Hart, 1987)	2.1	5.17
Continent radioactive heat production	(=36.6mW/m^2)	0.7	1.74
Mantle radioactive heat production	(=Earth – continental heat production)	1.4	3.43
Core heatflux into mantle	<10% of surface heat flow	<0.4	<1
Mantle thermal heat deficit (sum sources and sinks except slab potential energy)		<-1.2	<-2.9

Significant Present-day Radioactive Heat Sources.

	Heat	Decay Constant (Ga^{-1})	Halflife (Ga)
U(^{235}U,^{238}U)	9.71x10^{-5}W/kg	-2.6	4.1,0.7
^{232}Th	2.69x10^{-5}W/kg	1.2 [d=0.5]	9.8
K(^{40}K)	3.58x10^{-9}W/kg	-1.1	1.25

SI Prefixes										
n nano- 10^{-9}	μ micro- 10^{-6}	m milli- 10^{-3}	c centi- 10^{-2}	d deci- 10^{-1}	da deka- 10	h hecto 10^{2}	k kilo 10^{3}	M mega- 10^{6}	G giga- 10^{9}	T tera- 10^{12}

Useful Conversions:	Useful Constants + typical mantle value
1 year $\approx \pi \cdot 10^7$ sec	ideal gas constant $R = 8.314\, J/mol°K$
1 Ga $\approx \pi \cdot 10^{16}$ sec	heat capacity $c_p \approx 1100\, J/kg°K$ $\approx 3R/mole-atom$
1 cal = 4.18 J	heat of fusion $\Delta H^{fus} \approx R/mole-atom$
1 bar = 0.1 MPa=10^5Pa	thermal conductivity $k \approx 3\, W/m°K$
1 kbar = 0.1 GPa	thermal diffusivity $\kappa \approx 10^{-6}\, m^2/s$
1 atm = 0.101 MPa	thermal expansivity $\alpha \approx 3x10^{-5}{}°K^{-1}$
1 hfu = 47 mW/m^2	lithosphere density $\rho_{lith} \approx 3300 kg/m^3$
1 Angstrom = 10^{-10} m	continent density $\rho_{cont} \approx 2700\, kg/m^3$
1 ° = 60 miles $\approx 111 km$	mantle density $\rho_{man} \approx 5000\, kg/m^3$
	gravitational acceleration g $\approx 9.8\, N/kg$ (correct at Earth's surface; approximately correct through the entire mantle)

Useful Constants.

Variable	Value
mantle mass	$4.06x10^{24} kg$
continental crust mass	$2x10^{22} kg$
hydrosphere mass	$1.35x10^{21} kg$
core mass	$1.88x10^{24} kg$
mean Earth radius	6371 km
mean core radius	3486 km
mantle thickness	2880 km
mean continent crust thickness	35 km(±5)
mean ocean crust thickness	6 km(+1/-2)
mean continent tectosphere thickness (likely chemical + thermal boundary layer)	200 km (±50)
mean ocean lithosphere thickness	80 km(±10)
oceanic compositional lithosphere thickness	60 km (+20/-10)
Earth's surface area	5.1x10^{14} m^2
continental area (inc. cont. shelves)	2x10^{14} m^2
seafloor area (less cont. shelves)	3.1x10^{14} m^2