What is a fluid?
What is its rest state?

[problems 2, 4-11]

Lecture #1&2 Fluids at Rest

Intro – Importance of physical insight/intuition! → motion of fluids

Fluid mechanics is the study of how fluids move, and

The first question is: (1) what is a fluid? How would we distinguish (2) a liquid from a (3) gas? How do fluids behave when they are in contact. What is their (fluid) (4) rest state like?

These questions are the subject of the first chapter. Really very nice chapter and also quite an important one.

A. What is a fluid:

1. Something that has no defined form to which it relaxes when forces removed.

[Solid → liquid when exceed yield point
Rheology → (solid at short Δt, fluid at long)]

w/ fluids that remember shape

2. gas – expand to fill container

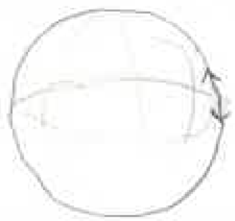
-vs-

liquid – volume not change much – fill bottom of a container and have air space above.

Difference has to do w self-attraction of molecules – strong in liquid, weaker in gas, different in different fluids

3. Immiscible fluids — have interface

w/ behaves as if it were in tension — logical — one fluid likely to have more tightly bound molecules than the other. If there is free case it will curl into a self-loving ball:



At the boundary there will be a net inward-directed force because of the curvature and the fact The interior fluid is more attracted to itself than it is to the outside fluid

Force balance dictates that

(1) $$\boxed{p_i - p_o = \frac{2\sigma}{R}}$$

where σ is the surface tension in N/m (force per unit length of a cut in the interface)

$$F = 2\pi r \sigma \qquad \equiv N$$

$$F = \pi r^2 (p_i - p_o)$$

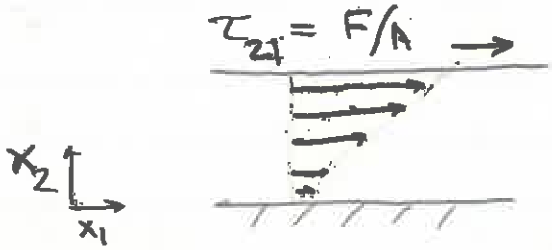
$$\therefore \; p_i - p_o = \frac{2\sigma}{r} \qquad N/m^2$$

plants — suction of 40 atm

interfacial

Surface tension is what draws water into capillary tube, explains wetting etc. It also ~~and it~~ controls flow of more than one fluid in porous media. — Fingering of flow into vadose zone (Impt for pesticides etc.) We'll get to them later in course.

Viscosity is the property of a liquid that determines how it will resist shear



unit outward normal

direction of force

(2) $$\boxed{\tau_{21} = \mu \frac{du}{dy}}$$

$\tau_{21} \equiv \frac{N}{m^2}$, $\frac{du}{dy} \equiv s^{-1}$

$$\therefore \mu \doteq \frac{N \cdot s}{m^2} = \frac{kg\, m\, s^{-2}\, s}{m^2} = kg\, m^{-1} s$$

μ = dynamic viscosity

$$\doteq kg\, m^{-1} s^{-1} \text{ or } \frac{N \cdot s}{m^2}$$

$$\frac{\mu}{\rho} \doteq \frac{kg\, m^{-1} s^{-1}}{kg/m^3} = m^2 s^{-1} \quad \text{(diffusion constant!)}$$

$\frac{\mu}{\rho} = \nu$ = kinematic viscosity, measures how velocity gradients in a fluid diffuse

Analog to Thermal diffusivity, κ

kinematic because just + trajectories

$$z = 2\sqrt{\kappa t}$$

distance ~~thermal front~~ diffuses into a solid

$$z = 2\sqrt{\nu t}$$

distance velocity discontinuity will diffuse into liquid (transition time of tyranic above to ss steam illustrated)

means only is the ability to maintain shear gradient hydrostatic v.then?

B. Fluids at Rest

What is the rest state of a fluid like?

VIMP question, and also a little non-trivial.

1. pressure in fluid will increase because of weight of overlying fluid.

neutrally stable fluid isentropic (+ ∴ also adiabatic)

physically P, T, ρ s.t. all same inside & outside an adiabatic socks

$\hat{g}$ ↓ $\hat{z}$ $\hat{x}$

vertically $$\frac{dp}{dz} = -\rho g$$

in general $$\nabla p = -\rho g \hat{z}$$

(3) IF no flow $$\boxed{\nabla p + \rho g \hat{z} = 0}$$

Dimensions of R

$$P = \rho R T$$

$$\frac{kg \cdot m \cdot s^{-2}}{m^2} = \frac{kg}{m^3} R \; K$$

$$R \overset{m}{=} m^2 s^{-2} K^{-1} \checkmark$$

$$= \frac{R_0}{M_{air}} \overset{m}{=} \frac{J \; \cancel{kmol^{-1}} \; K^{-1}}{kg \; \cancel{kmol^{-1}}} = \frac{(\cancel{kg} \, m \, s^{-2} \cdot m) K^{-1}}{\cancel{kg}}$$

$$\checkmark = m^2 s^{-2} K^{-1}$$

$$R_0 = 8314.36 \; J \, kmol^{-1} K^{-1}$$

$$= 2 \; kcal \; kg^{-1} K^{-1}$$

cal = 4.186 J

$$R_{air} = \frac{8314.36 \; J \; \cancel{kmol^{-1}}}{28.966 \; kg \; \cancel{kmol^{-1}}} K^{-1}$$

$$= 287 \; \frac{\cancel{kg} \, m \, s^{-2} \cdot m \; K^{-1}}{\cancel{kg}}$$

$$\boxed{m^2 s^{-2} K^{-1}}$$

$$P = \tfrac{1}{3} \rho \bar{v}^2 = \tfrac{1}{3} m \, n \, \bar{v}^2$$

$\boxed{\text{\# of part. per unit volume}}$

$$c_v = \frac{1}{n} \frac{\partial U}{\partial T}\Big|_v = \frac{1}{n} \frac{\partial}{\partial T} \left(\tfrac{3}{2} n R T \right) = \tfrac{3}{2} R$$

$$\left\{ \begin{array}{l} \tfrac{1}{2} m \bar{v}^2 = \tfrac{3}{2} \sigma T \\ U = \tfrac{3}{2} N \sigma T = \tfrac{3}{2} n R T \end{array} \right.$$

Boltzmann's const $= R/N_0$

(3a)
$$\begin{pmatrix} \frac{\partial p}{\partial x} \\ \frac{\partial p}{\partial y} \\ \frac{\partial p}{\partial z} \end{pmatrix} = -\rho \begin{pmatrix} 0 \\ 0 \\ g \end{pmatrix}$$

Now
$$p = -g \int_{z_s}^{z} \rho \, dz$$

and for $\partial_x p = \partial_y p = 0$, $\rho = \rho(z)$ only

Otherwise p would vary laterally

No flow iff $p_0(z) = -g \int \rho_0(z)\, dz$

where ρ_0 is the no-flow density of fluid and a function of z only. p_0 is also only a function of z. p_{00} might be Atm. pressure at surface of E for example.

So $\rho = \rho(z)$ for $u = 0$

Meaning of R

$m^2 s^{-2} K^{-1}$

For a perfect gas at constant T,

Gas law

(4) $\boxed{pV = RT \quad \text{or} \quad p = \rho R T, \text{ so}}$

$\frac{N}{m^2} = \frac{kg\, m\, s^{-2}}{m^2}$ $\quad m^3/kg \quad$ $m^2 s^{-2} K^{-1}$ $\quad$ K

$m^2 s^{-2} = m^2 s^{-2}$

$$\frac{dp}{dz} = -\rho g = -\frac{p g}{RT}$$

for ideal gas

Exercise

$$J = F \cdot dx$$

$$kg\, m\, s^{-2} \cdot m$$

$$\overset{m}{=} kg\, m^2 s^{-2}$$

$$R \overset{m}{=} \frac{J\, \cancel{kmol^{-1}}\, K^{-1}}{kg\, \cancel{kmol^{-1}}} \overset{m}{=} \frac{J\, K^{-1}}{kg}$$

$$\overset{m}{=} \frac{\cancel{kg}\, m^2 s^{-2} K^{-1}}{\cancel{kg}} = m^2 s^{-2} K^{-1}$$

$$\frac{RT}{g} \overset{m}{=} \frac{\cancel{m^2}\, \cancel{s^{-2}}\, \cancel{K^{-1}}\, \cancel{K}}{\cancel{m}\, \cancel{s^{-2}}} \overset{m}{=} m$$

$$\frac{RT}{g} = \frac{(287)(250)}{9.8} = \delta \cong 7.3\ km$$

$$\int_{p_s}^{p} \frac{dp}{p} = \int_{z=0}^{z} -\frac{g}{RT}\, dz$$

$$\ln p = -gz/RT$$

(5)

$$\boxed{p = p_s\, e^{-gz/RT} = p_s\, e^{-z/\delta} \qquad \delta = RT/g = \text{scale height of the atmosphere.}}$$

units of R - see p. 5 - shows RT/g = L

at $T = 250°K$ $(-23°C)$

$g = 9.8\ m/s^2$

$R = 287$ ($= 8314.36\ J\ kmol^{-1}\ K^{-1} / m_{dry\ air}$; $m_{dry\ air} = 28.996\ kg/kmol$)

R_0 — parenthetically

$\frac{250 \cdot 287}{9.8} = 7{,}300\ m$

shows units are correct

(5a) $\boxed{\delta = 7.3\ km}$ — P drops to p_s/e at 7.3 km; ρ drops to ρ_s/e @ 7.3 km

C. More Realistic Atmosphere

Of course $T_{atm} \neq$ constant $-23°C$,

so the situation is a bit more complicated. In fact this

gets tricky, and we must understand some thermodynamics

We will not do justice to the thermodynamics in our discussion here.

But we do need to understand the fundamentals of what constituted an atmosphere with rest (adiabatic and isentropic). (freshmen)

Thermodynamics is a fundamental science — the heart of physics, chemistry + engines (carnot cycle). You all know 1st law = conservation of energy, 2d law = behavior of entropy (heat flows from hot to cold), 3d law absolute Zero. Also fundamentally related to mathematics (or vice versa). Every time you return to thermodynamics in life you will learn a bit more + appreciate it more. Let's take another run.

1. Prolog.

Let's suppose that the vibrational + rotational + translational energy of the molecules of a gas represents its internal energy. (More could be involved — magnetic, chemical etc — but for now suppose it is just kinetic.) The internal energy can be thought of as approximately T, but unlike T it is an extensive parameter. That is independent of how the internal energy was acquired and proportional to the amount of fluid at that energy. For example if 1 kg has e internal energy, 2 kg has 2e, ~~etc~~

Now let's say that e is a function of a bunch of variables like entropy and volume (and the electrochemical potential of each component of the fluid, and ...),

so

System w/ rigid body of ideal heat compression + mechanical work can be done on the system

$$e = e(s, V)$$

$$de = \left(\frac{\partial e}{\partial s}\right)_V ds + \left(\frac{\partial e}{\partial V}\right)_s dV \qquad (6)$$

If we define

$$T \equiv \left(\frac{\partial e}{\partial s}\right)_V$$

$$P \equiv -\left(\frac{\partial e}{\partial V}\right)_s$$

Then defined is s.t. de is a perfect differential - integral depends only on end pts equal'ff quasi-static, reversible (frictionless), adiabatic

adiabatic + frictionless (quasi-static or reversible) = isentropic $\Delta s = 0$

Then

(5) $$de = T\,ds - P\,dV$$

First Law / Thermodynamics (Conservation of energy)

need only know initial + final states to get energy flux into the system

$$\underset{\text{quasistatic heat flux into the system}}{\underbrace{T\,ds}_{\equiv\, đQ}} \qquad \underset{\text{quasi-static work done on the fluid by the environment}}{\underbrace{-P\,dV}_{=\, đW_m}}$$

entropy can only increase

(do work infinitely slowly so it is reversible)

$$đQ = de - đW_m = de + P\,dV$$

imperfect differentials $đQ$

"The integrals of $đW_m$ and $đQ$ for a particular process are the work and heat fluxes in that process; The sum is the energy difference Δe, which alone is independent of the process." (p20 Callen)

For perfect, frictionless engine, $de = 0$,

and T can be converted to W as determined

by

$$T\,ds - P\,dV = 0$$

$$đQ + đW = 0$$

add heat, get work.

If fluid driven cyclic.

$$đQ = -đW$$

work done by environment

work done by fluid

Exact Differential

$$de = M(x,y)\,dx + N(x,y)\,dy$$

exact iff
$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$de = \underbrace{\left(\frac{\partial e}{\partial s}\right)_V}_{\equiv T} ds + \underbrace{\left(\frac{\partial e}{\partial V}\right)_s}_{\equiv -P} dV$$

$$de = T\,ds - P\,dV$$

exact iff
$$\left.\frac{\partial T}{\partial V}\right|_s = -\left.\frac{\partial P}{\partial s}\right|_V$$

$$\|$$

$$\left(\frac{\partial^2 e}{\partial s \partial v}\right)_{vs} = \left(\frac{\partial^2 e}{\partial s \partial v}\right)_{vs}$$

<u>clearly</u> exact criterion satisfied

because T and P are <u>defined</u> s.t. it is!

Lecture # 2 – Fluids at Rest: The Thermodynamics

Last time we ended with a brief discussion of a thermodynamic system with flexible boundaries through which heat can pass and mechanical ($p \Delta V$) work can be done on the system.

We noted that state variables (variables that depend only on the local conditions (p, T, etc.) and not on the path to that point) are the foundation of thermodynamics

state variable ≡ exact differential ≡ perfect differential etc.
≡ extensive parameter

Consider internal energy, e:

$$e = e(s, v)$$

$$de = \underbrace{\left(\frac{\partial e}{\partial s}\right)_v}_{\equiv\, T} ds + \underbrace{\left(\frac{\partial e}{\partial v}\right)_s}_{\equiv\, -P} dv$$

(c) $$de = \underbrace{T\,ds}_{\equiv\, đq} \; \underbrace{- P\,dv}_{\equiv\, đw_m}$$

heat input – $đq$ $\qquad$ $đw_m$ – mechanical work

REVIEW
2nd law
$dq \le T ds$
= iff reversible

designed to be exact or perfect differential because criterion for exact differential is:

$$\left(\frac{\partial T}{\partial v}\right)_s = \left(\frac{\partial(-P)}{\partial s}\right)_v$$

$$\left(\frac{\partial^2 e}{\partial s\, \partial v}\right)_{sv} = \left(\frac{\partial^2 e}{\partial s\, \partial v}\right)_{sv}$$

So de is exact differential, so $\int_1^2 de = e\Big|_1^2 = e_2 - e_1$

internal energy

$$de = T\,ds - p\,dv$$

$$\left(\frac{\partial T}{\partial v}\right)_s = \left(\frac{\partial p}{\partial T}\right)_v$$

maxwell relation — criterion for exact diff'l, e is exact.

Now we can define other quantities that are state variables and get their maxwell relations

Helmholtz free energy $\quad f = e - Ts$
(max E avail at T=const)

enthalpy $\quad h = e + Pv$
(max E avail at P=const)

Gibbs free E $\quad g = e - Ts + Pv$
(max E avail at P,T = const)

So for example:

enthalpy

$$d(\underbrace{e}_{Tds - pdV = de} + Pv) = dh(s,P) = T\,ds + v\,dp$$

maxwell relation $\quad \left(\frac{\partial T}{\partial p}\right)_s = \left(\frac{\partial v}{\partial s}\right)_p$

Helmholtz free Eg

$$d(\underbrace{e}_{= Tds - pdV = de} - Ts) = df(T,v) = -s\,dT - p\,dv$$

Maxwell Relation $\quad \left(\frac{\partial s}{\partial v}\right)_T = \left(\frac{\partial p}{\partial T}\right)_v$

$= Tds - p\,dv = de$

Gibbs free Eng

$$d(e - Ts + pv) = dg(T,p) = -s\,dT + v\,dp$$

$$-\left(\frac{\partial s}{\partial p}\right)_T = \left(\frac{\partial v}{\partial T}\right)_p$$

Maxwell's relations allow any second derivative to be expressed as a combination of c_v, c_p, α, and K_T where

$de = T\,ds - p\,dv$

$(de)_v = T\,ds = dq$

$$\left(\frac{\partial e}{\partial T}\right)_v = \left(\frac{dq}{dT}\right)_v$$

$\equiv c_v$

$$C_v = \left(\frac{\partial e}{\partial T}\right)_v = \left(\frac{dq}{dT}\right)_v$$

heat capacity const vol

$$C_p = \left(\frac{\partial h}{\partial T}\right)_p = \left(\frac{dq}{dT}\right)_p$$

heat capacity const p

$$K_T \equiv \frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T$$

isothermal compressibility

$dh = T\,ds + v\,dp$

$$\left(\frac{\partial h}{\partial T}\right)_p = T\left(\frac{\partial s}{\partial T}\right)_p = \left(\frac{dq}{dT}\right)_p$$

$\equiv c_p$

$T\,ds = dq$

$$\alpha = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p$$

coeff thermal expansion

specific vol (vol/kg)

Maxwell's relations can be used to show, for example, that:

① $\boxed{C_v = c_p - T v\,\alpha^2/K_T}$

② $h \equiv e + pv$

$dh = de + d(pv)$

$\boxed{c_p\,dT = c_v\,dT + R\,dT}$

perfect gas

$$C_p - C_V = R$$

last time indicated $C_v = \frac{3}{2} R$ for monatomic gas

$C_v = \frac{5}{2} R$ for diatomic gas

$$C_V = \frac{f}{2} R$$

$$C_p = \left(\frac{f}{2} + 1\right) R$$

OK – Now back to the rest state of a fluid —
dynamically

last time saw that a neutrally stable atmosphere is isenthalpic (which is also adiabatic)

(7) second law "heat flows downhill"

$$\boxed{dq \leq T\, ds}$$

$= T\, ds$ if reversible process

$$\int_1^2 ds \geq \int_1^2 \frac{dq}{T}$$

$= 0$ if adiabatic ($dq = 0$)

$= 0$ if S same everywhere (isentropic)

$\frac{\partial p}{\partial z} = \rho g$ ← adiabatic driving rock

ρ, T, P same everywhere = adiabatic, neutral rest state

Neutral atmosphere is adiabatic, isentropic and the two descriptions may mean the same thing

For an adiabatic ($dq = 0$) and isentropic ($ds = 0$) fluid

Internal E $\quad T\,ds = de + P\,dv$

entropy $\quad T\,ds = dh - v\,dp$

$0 = dq$ Isentropic or adiabatic

all state variables so holds for any fluid — also valid for reversible processes

(10)

$$de \equiv (c_v\,dT)_v = -P\,dv$$

$$dh \equiv (c_p\,dT)_p = v\,dp$$

dividing

Perfect gas

$$\gamma = \frac{c_p}{c_v} = \frac{(f/2+1)R}{(f/2)R}$$

$$\gamma = \frac{f+2}{f}$$

$$\frac{c_p}{c_v} \equiv \gamma = \frac{v\,dp}{-P\,dv}$$

$$\frac{dP}{P} = -\gamma\frac{dv}{v} = \gamma\frac{d\rho}{\rho}$$

$$\ln p = \ln\rho^{\gamma} + \text{const}$$

$$\ln\frac{p}{\rho^{\gamma}} = \text{Const}$$

(11) $$\boxed{\frac{p}{\rho^{\gamma}} = \text{Const}}$$

Now for a perfect gas $p = \rho R T$, so:

$\rho \propto p^{1/\gamma}$

(14)
$$\boxed{\frac{T_1}{T_2} = \frac{(p/\rho)_1}{(p/\rho)_2} = \frac{p_1^{1-\frac{1}{\gamma}}}{p_2^{1-1/\gamma}} = \left(\frac{p_1}{p_2}\right)^{1-1/\gamma}}$$

From (13), $\rho \propto p^{1/\gamma}$, it is also true that

(15)
$$\boxed{\frac{\rho_1}{\rho_2} = \left(\frac{p_1}{p_2}\right)^{1/\gamma}}$$

$$V \propto p^{\gamma}$$

$$\frac{V_1}{V_2} = \left(\frac{p_1}{p_2}\right)^{\gamma} \qquad \frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{1-1/\gamma}$$

flow for HW

$$\frac{p_1}{p_2} = \left(\frac{V_1}{V_2}\right)^{1/\gamma} \qquad \frac{\gamma-1}{\gamma}$$

$$\left(\left(\frac{V_1}{V_2}\right)^{1/\gamma}\right)^{1-1/\gamma}$$

$$\shortparallel$$

$$\boxed{\frac{T_1}{T_2} = \frac{V_1}{V_2}}$$

4. The adiabatic temperature gradient dT_a/dz ⑦ (19)

from (14) → so what?

$$\ln T_a - \ln T_0 = (1 - 1/\gamma)(\ln p_a - \ln p_0)$$

$$\frac{1}{\cancel{T_a}}\frac{dT_a}{dz} = \frac{\gamma - 1}{\gamma}\,\frac{1}{p_a}\,\frac{dp_a}{dz}$$

$$\frac{c_p/c_v - 1}{c_p/c_v} \qquad \rho_a R \cancel{T_a} \qquad -\rho_a g$$

$$\downarrow$$

$$c_p - c_v$$

now know how P, T, ρ should vary in a proper adiabatic (well-stirred) isentropic (rest state) atmosphere

$$\frac{c_p - c_v}{c_p}\,\frac{1}{c_p - c_v} = \frac{1}{c_p}$$

Remember: $\frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{1-1/\gamma}$ (p14)

$$\boxed{\frac{dT_a}{dz} = \frac{-g}{c_p} = \Gamma_a} \qquad (16)$$

adiabatic lapse rate

So

$$\Gamma_a = \left(\frac{\gamma - 1}{\gamma}\,\frac{1}{p_a}\,\frac{dp_a}{dz}\right)T_a$$

$$c_p = 1005 \text{ J kg}^{-1}\text{ K}^{-1}$$

$$g = 9.8 \text{ m s}^{-2}$$

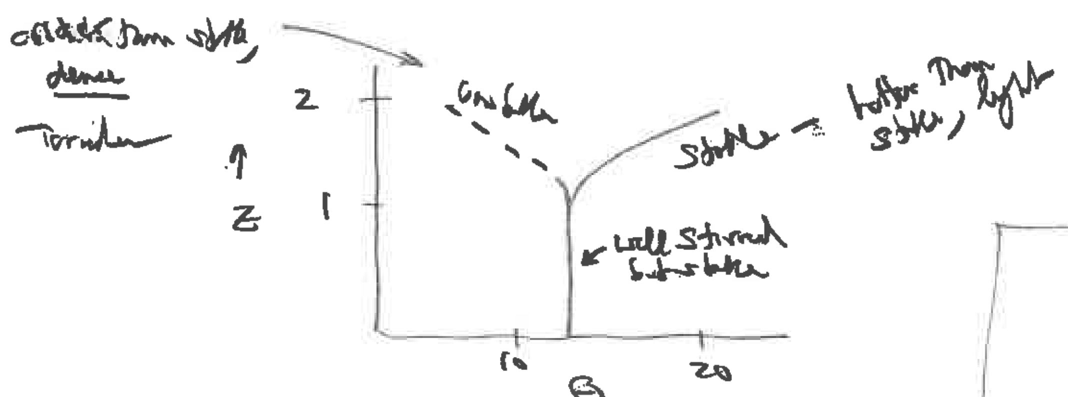
$$\frac{-g}{c_p} \cong 10^{-2}\,°\text{K/m} = 10°\text{C/km}$$

a. Potential Temperature, θ, and stability of the atmosphere

It is customary to correct T for adiabatic effects by determining what that T would be at sea level if it were adiabatically moved to sea level. This sea level temperature is called potential temperature + indicated by θ.

from (14) $$\frac{\Theta}{T} = \left(\frac{p_s}{p}\right)^{1-1/\gamma}$$

(17) $$\boxed{\Theta = T\left(\frac{p_s}{p}\right)^{1-1/\gamma}}$$ Potential T of Atmosphere



$$\boxed{\frac{\partial\theta}{\partial z} < 0 \quad \text{unstable}}$$

b. Potential Density in Oceans

$$\frac{\rho}{\rho_\theta} = \left(\frac{p}{p_s}\right)^{1/\gamma}$$

from (15)

θ = equivalent temp at surface

$$\rho = \rho_\theta \left(\frac{p}{p_s}\right)^{1/\gamma} \qquad (18)$$

$$\boxed{\rho_\theta = \rho\left(\frac{p_s}{p}\right)^{1/\gamma}}$$

$$\boxed{\frac{\partial \rho_\theta}{\partial z} > 0 \quad \text{unstable}}$$

$\rho = \rho(T)$ monitor $T(t)$ measure!

c. Velocity of sound

for oceans

One quick and useful digression. Sound pressure variations are so rapid heat has no time to escape. Thus for sound the atmosphere (or ocean, or mantle) is adiabatic. What is the speed of sound in a

Speed of sound is:

$$c^2 = \left(\frac{\partial p}{\partial \rho}\right)_s \longleftarrow \text{const entropy}$$

$N\,m^{-2}$
$kg\,s^{-2}\,m\,m^{-2}$ $\quad \frac{kg\,s^{-1}\,m^{-1}}{kg\,m^{-3}} \rightarrow m^2 s^{-2}$

from (P10) $\quad \frac{V\,dp}{-p\,dV} = \gamma = \frac{\rho}{p}\frac{dp}{d\rho}$ at const s

$$\frac{dV}{V} = -\frac{d\rho}{\rho} \qquad \frac{dp}{d\rho} = \frac{\gamma p}{\rho} = \frac{\gamma \cancel{\rho} RT}{\cancel{\rho}}$$

For perfect gas $\quad p = \rho RT$

Then $\quad \boxed{c = \sqrt{\gamma RT}}$ for perfect gas

For water

$$c = \sqrt{B/\rho}$$

B = bulk modulus

$$\equiv \frac{-1}{\frac{1}{V}\frac{dV}{dp}} = \frac{-V\,\Delta p}{\Delta V}$$

$$\overset{M}{\equiv} N/m^2 = \frac{kg\text{-}m\text{-}s^{-2}}{m^2}$$

$$B/\rho \equiv \frac{\frac{kg\text{-}m\text{-}s^{-2}}{m^2}}{kg/m^3} \equiv m^2/s^2$$

$$\therefore \sqrt{\frac{B}{\rho}} = \text{velocity}$$

Make it measure of climate change

°C	v (m/s)
0	1403
10	1447
50	1541
90	1550

HW = capillary, viscosity, lots of use of γ, Thermodynamics