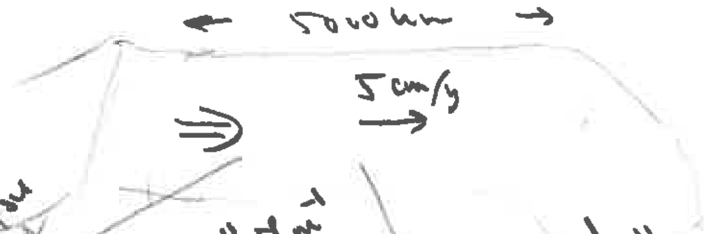
Lecture 22

A. Forces Required to Drive Plates

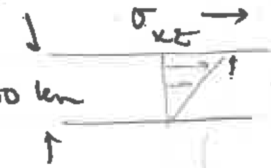


What force per unit length of trench + Ridge is required to drive a large plate at 5 cm/y?

Bottom line
$F_{needed} = 7.9 \times 10^{11}$ N m^{-1}
Trench pull $= 378 \times 10^{11}$
ridge push $= 33.6 \times 10^{11}$
plume push $\cong 0$

plenty to push! self regulating

F/m length



$\mu = 10^{19}$ pa·s

$v_x = 5$ cm/yr

$$\sigma_{xz} = \mu \frac{\partial v_x}{\partial z} = 10^{19} \frac{.05/3.15 \times 10^7}{100 \times 10^3 \text{ m}}$$

$$= 1.58 \times 10^5 \text{ N/m}^2$$

For a 5000 km long plate the Force required is

$$F_{plate} = (5000 \times 10^3)(1.58 \times 10^5 \text{ N/m}^2) =$$

$$= 7.93 \times 10^{11} \text{ N m}^{-1}$$ ← force per unit width of plate

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Are there forces of this magnitude available?

just state – skip to p 14

– Trench Pull

– Ridge push

First consider trench pull – (1) need good estimate of plate density.

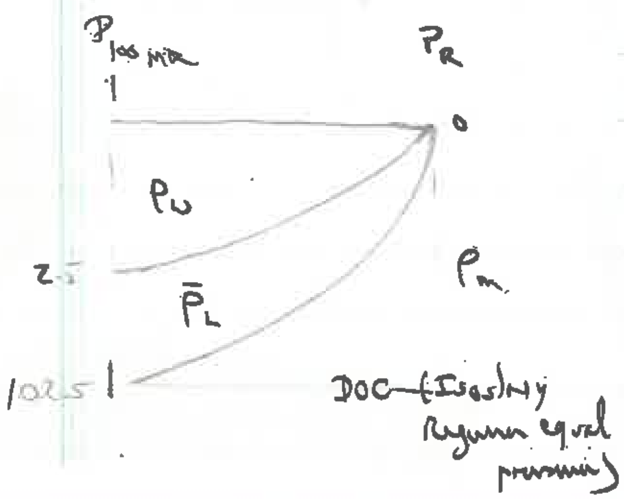
That's all that!

B. Density of the Lithosphere

Isostasy requires that the pressure at the depth of compensation (DoC) under 100 Ma plate be the same as at the ridge:

plate 100 km thick + 2.5 km below SL

Then:



$$P_{100\,Ma}(102.5\text{ km}) = P_{ridge}(102.5\text{ km})$$

$$P_{Ridge}(z=102.5) = \rho_m g z = (3170)(10)(102.5\times10^{3})$$

$$= 3249 \text{ MPa}$$

$$P_L(z=102.5) = 3249 \text{ MPa}$$

$$= \overset{1000}{\rho_w}\,\overset{10}{g}\,(2500\text{ m}) + \bar{\rho}_L\, g\, \overbrace{100\times10^{3}\text{ m}}^{L}$$

$$\therefore \boxed{\bar{\rho}_L} = \frac{(3249 - 25)\times10^{6}\text{ Pa}}{10\cdot 100\times10^{3}} = \boxed{3{,}224 \text{ kg/m}^3}$$

$\Delta\rho = \rho_0 \alpha K$

$\alpha = 2\times10^{-5}\ K^{-1}$

cools 600°C

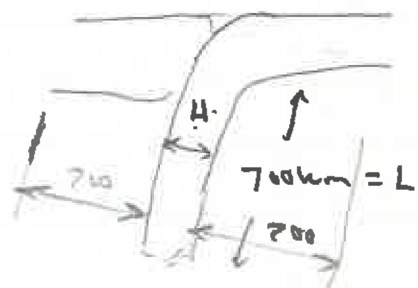
$\Rightarrow \Delta\rho = 1.2\%$

CONDUCTIVE COOLING TRICK! Look at [illegible]

C. Trench Pull......

Earthquakes persist in subducting oceanic lithosphere to ~700 km depth. If we approximate the trench pull as the excess mass in the ocean lithosphere from the seafloor to 700 km depth times g, then:

$$F_{\substack{Tr\\pull}} = \Delta m \cdot g$$



$$= \left(\bar{\rho}_L - \rho_m\right) H L g$$

$$= (3224 - 3170)(100 \times 10^3\ m)(700 \times 10^3\ m)(10)$$

$$\boxed{F_{\substack{Tr\\pull}} = 3.78 \times 10^{13}\ N\,m^{-1}}$$

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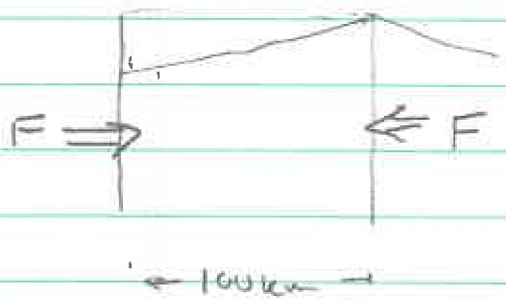
D Ridge Push



→ 110 km from lemma

$\rho_m = 3170\ kg/m^3$

$g = 10$

plate = 110 km thick

$\rho_{11km} = 3224\ kg/m^3$

Force Balance:

At Ridge

$$P_R = \rho_m g z$$

$$F_r = \int_0^{110} \rho_m g z\, dz = \frac{\rho_m g z^2}{2}\Big|_0^{110\ km}$$

$$= 1.950 \times 10^{14}\ N m^{-1}$$

At 100 my

$$F_{100\ my} = \int_0^{z=2.5} \rho_w g z\, dz + \int_{z=2.5}^{z=102.5} \bar{\rho}_L g z\, dz$$

$$= \frac{\rho_w g z^2}{2}\Big|_0^{2500} + \frac{\bar{\rho}_L g z^2}{2}\Big|_{2500}^{103500}$$

$$= 3.12 \times 10^{10} + 1.917 \times 10^{14}$$

$$= 1.917 \times 10^{14}$$

1.950
−1.917
.033 × 10^14

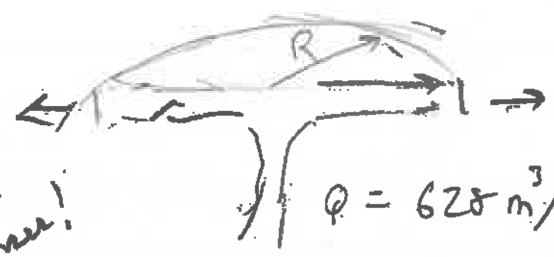
Ridge Push $\boxed{F_R - F_{100\ my} = 3.36 \times 10^{12}\ N m^{-1}}$

Turcotte + Schubert get $3.9 \times 10^{12}\ N m^{-1}$

E. Plume Push –

– Quite small + tends to cancel

A better approach might be to distribute the plume delivery rate radially such that the average velocity in a 200 km thick asthenosphere would be



mantle plumes + plate convection separate processes!

$$Q = 628\ m^3/sec$$

$$= 19\ km^3/yr$$

$$\bar{V}_r = \frac{Q}{2\pi R H} = \frac{19\ km^3/yr}{2\pi H R} = \frac{1.5 \times 10^{-2}}{R(km)}\ \frac{km}{yr}$$

$$\bar{V}_r \underset{R=2000\,km}{=} \boxed{0.7\ cm/yr}$$

Needed to push plate ~	$7.9 \times 10^{11}\ Nm^{-1}$
Trench pull	$3.8 \times 10^{13}\ Nm^{-1}$
Ridge push	$3.3 \times 10^{12}\ Nm^{-1}$

∴ Plenty of force to drive plates ⇒ self-regulation of solid

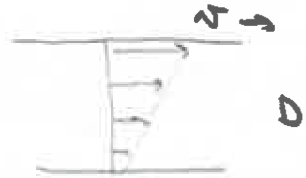
⇓ from p. 9

STARTUP OF PLATES

units $\quad \nu = \mu/\rho \qquad \mu \frac{U_e}{D} = F/A$ 5

$\uparrow \quad \stackrel{m}{=} m^2/sec \qquad N/m^2 \qquad N/m^2$

viscous diffusivity



$$\eta \frac{v}{D} = F/A$$

$N \stackrel{m}{=} kg\, m\, s^{-2} \qquad \frac{N\text{-}s}{m^2} \qquad N/m^2$

$$\nu \equiv \frac{\eta}{\rho} = \frac{\cancel{kg}\, s^{-1}\, m^{-1}}{\cancel{kg}/m^3} \stackrel{m}{=} m^2/sec$$

C. Estimate of Plate Startup time

In fact $z = 2\sqrt{\nu t}$ is a very useful relationship. Suppose we break the lithosphere and initiate a new plate motion. How long will it take for a steady shear profile to become established in the asthenosphere?

$$z = 2\sqrt{\nu t} \; ; \quad \boxed{t = \frac{z^2}{4\nu}}$$

if $\nu = \dfrac{10^{19}\,\text{Pa·s}}{3170\ \text{kg/m}^3}$ and $z = 200$ km

$$t = \frac{(2\times10^5)^2}{4\nu} = \frac{4\times10^{10}}{3\times10^{15}} = 1.3\times10^{-5}\ \text{sec}$$

Clearly the shear profile becomes established very fast.

If we consider water, where $\mu = 10^{-3}$ Pa·s

and $\rho = 1000$, an imposed velocity at the top surface

of a 1 m deep fish tank will take ~3 days to establish

$$t = \frac{z^2}{4\nu} = \frac{1}{4\ 10^{-6}} = 0.25\times10^{6}\ \text{sec}$$

1m

$$= 2.89\ \text{days}$$

a uniform shear flow. If the surface shear changes

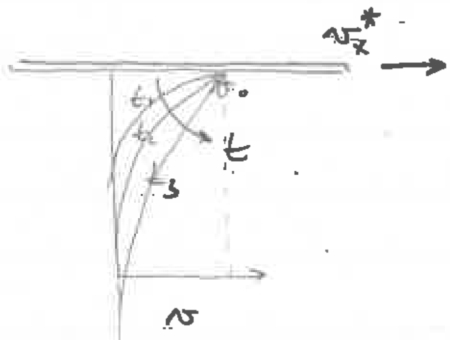
faster than this, only the near-surface fluid will ever

be effected. Flow will be confined to a

"boundary layer" near the surface.

Lemma

Equation Derivation for Kundu + Cohen
6. Plate Startup Transient



Kundu p. 289 shows the equation describing the transient in velocity for increasing the velocity of an infinite plate over an infinite half-space of fluid from 0 to v_0 at $t=0$ is exactly the same as for the increase in temperature of an infinite solid half-space whose surface is suddenly changed from 0 to T_s at $t=0$.

$$v_x = \text{constant in } x \text{ direction}$$

$$\underline{\nabla} \cdot \underline{v} = \cancel{\frac{\partial v_x}{\partial x}} + \frac{\partial v_y}{\partial y} = 0$$

$$\therefore \frac{\partial v_y}{\partial y} = 0 \quad \text{and} \quad v_y = 0 \text{ everywhere because } v_y(y=0) = 0$$

The Navier stokes equation is

$$\rho \frac{\partial v_x}{\partial t} = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 v_x}{\partial y^2} .$$

But v_x cannot vary in x and therefore $\frac{\partial p}{\partial x} = 0$.

So

$$\frac{\partial v_x}{\partial t} = \frac{\mu}{\rho} \frac{\partial^2 v_x}{\partial y^2} = \nu \frac{\partial^2 v_x}{\partial y^2}$$

dynamic viscosity

kinematic viscosity $\stackrel{m}{=}$ m^2/sec

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial t^2}$$

Thermal Diffusivity $\stackrel{m}{=}$ m^2/sec

a. Solution 1:

Kundu + Cohen (p 289) show the solution is

$$\frac{v_x}{v_x^*} = 1 - \mathrm{erf}\left(\frac{y}{2\sqrt{\nu t}}\right)$$

, where erf is the error function integral

$$\mathrm{erf} \equiv \frac{2}{\sqrt{\pi}} \int_0^{\eta} e^{-\eta^2} d\eta$$

v_x^* is the velocity imposed at the top surface

and $\eta = y/2\sqrt{\nu t}$. Then method is to convert the second order partial differential equation in t and y to an ordinary differential equation in η. This is possible because there is no scale of t or y imposed by the boundary conditions or definition of the problem. Thus all curves of η vs. $^{v_x}/_{v_*}$ is a single curve and the velocity-depth profiles are self-similar (the same at all times, if scaled appropriately). Kundu + Cohen give a very nice derivation of the solution to this fundamental problem. Read p 288 - 293 carefully!

b: Physical Solution

Physical insight is provided by a simple conservation of energy approach. Consider the cooling of an initially hot half space.

This is from 201 notes