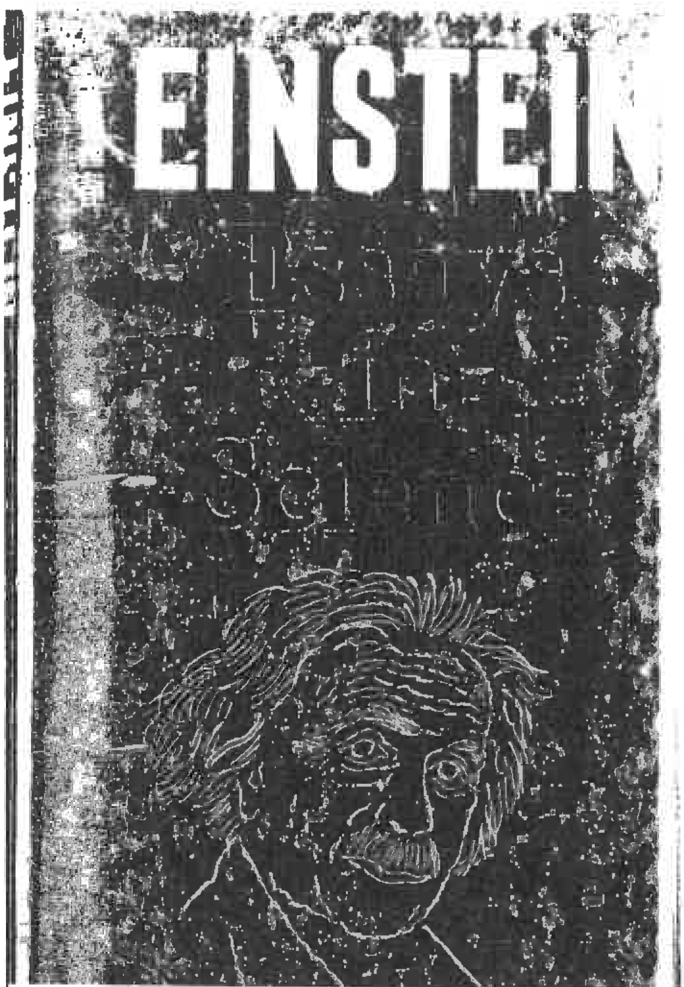


THE CAUSE OF THE FORMATION OF MEANDERS IN THE COURSES OF RIVERS AND OF THE SO-CALLED BEER'S LAW

It is common knowledge that streams tend to curve in serpentine shapes instead of following the line of the maximum declivity of the ground. It is also well known to geographers that the rivers of the northern hemisphere tend to erode chiefly on the right side. The rivers of the southern hemisphere behave in the opposite manner (Beer's law). Many attempts have been made to explain this phenomenon, and I am not sure whether anything I say in the following pages will be new to the expert; some of the relevant considerations are in any case known. Nevertheless, having found nobody who thoroughly understood the elementary principles involved, I think it is proper for me to give the following short qualitative exposition of them.

First of all, it is clear that the erosion must be stronger the greater the velocity of the current where it touches the bank in question, or the more steeply it falls to zero at any particular point of the confining wall. This is equally true under all circumstances, whether the erosion depends on mechanical or on

physico-chemical factors (decomposition of the ground). We must concentrate our attention on the circumstances which affect the steepness with which the velocity falls at the wall.

In both cases the asymmetry in relation to the fall in velocity in question is indirectly due to the occurrence of a circular motion to which we will next direct our attention. I begin with a little experiment which anybody can easily repeat.

Imagine a flat-bottomed cup full of tea. At the bottom there are some tea leaves, which stay there because they are rather heavier than the liquid they have displaced. If the liquid is made to rotate by a spoon, the leaves will soon collect in the center of the bottom of the cup. The explanation of this phenomenon is as follows:—The rotation of the liquid causes a centrifugal force to act on it. This in itself would give rise to no change in the flow of the liquid if the latter rotated like a solid body. But in the neighborhood of the walls of the cup the liquid is restrained by friction, so that the angular velocity with which it circulates is less there than in other places near the center. In particular, the angular velocity of circulation, and therefore the centrifugal force, will be smaller near the bottom than higher up. The result of this will be a circular movement of the liquid of the type illustrated in fig. 1. which goes on increasing until, under the influence of ground friction, it becomes stationary. The tea leaves are swept into the center by the circular movement and act as proof of its existence.

The same sort of thing happens with a curving stream (fig. 2). At every section of its course, where it is bent, a centrifugal force operates in the direction of the outside of the curve (from A to B). This force is less near the bottom, where the speed of the current is reduced by friction, than higher above the bottom. This causes a circular movement of the kind illustrated in the diagram. Even where there is

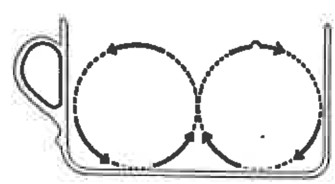


Fig. 1.

no bend in the river, a circular movement of the kind shown in fig. 2 will still take place, if only on a small scale and as a result of the earth's rotation. The latter produces a Coriolis-force, acting transversely to the direction of the current, whose right-hand horizontal component amounts to $2\, v\, \Omega \sin \phi$ per unit of mass of the liquid, where v is the velocity of the current, Ω the speed of the earth's rotation, and ϕ the geographical latitude. As ground friction causes a diminution of this force towards the bottom, this force also gives rise to a circular movement of the type indicated in fig. 2.

After this preliminary discussion we come back to the question of the distribution of velocities over the cross section of the stream, which is the controlling factor in erosion. For this purpose we must first realize how the (turbulent) distribution of velocities takes place and is maintained. If the water which was previously at rest were suddenly set in motion by the action of an evenly diffused accelerating force, the distribution of velocities over the cross section would

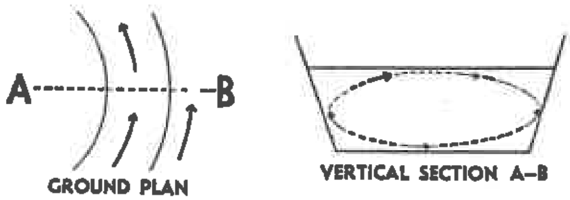


FIG. 2.

be even at first. A distribution of velocities gradually increasing from the confining walls towards the center of the cross section would only establish itself after a time, under the influence of friction at the walls. A disturbance of the (roughly speaking) stationary distribution of velocities over the cross section would only gradually set in again under the influence of fluid friction. Hydrodynamics explains the process by which this stationary distribution of velocities is established in the following way. In a systematic distribution of current (potential flow) all the vortex-filaments are concentrated at the walls.

They detach themselves and slowly move towards the center of the cross-section of the stream, distributing themselves over a layer of increasing thickness. The drop in velocity at the containing walls thereby gradually diminishes. Under the action of the internal friction of the liquid the vortex filaments in the inside of the cross section gradually get absorbed, their place being taken by new ones which form at the wall. A quasi-stationary distribution of velocities is thus produced. The important thing for us is that the adjustment of the distribution of velocities till it becomes stationary is a slow process. That is why relatively insignificant, constantly operative causes are able to exert a considerable influence on the distribution of velocities over the cross section. Let us now consider what sort of influence the circular motion due to a bend in the river or the Coriolis-force, as illustrated in fig. 2, is bound to exert on the distribution of velocities over the cross section of the river. The particles of liquid in most rapid motion will be farthest away from the walls, that is to say, in the upper part above the center of the bottom. These most rapid parts of the water will be driven by the circular motion towards the right-hand wall, while the left-hand wall gets the water which comes from the region near the bottom and has a specially low velocity. Hence in the case depicted in fig. 2 the erosion is necessarily stronger on the right side than on the left. It should be noted that this explanation is essentially based on the fact that the slow circulating movement of the water exerts a considerable influ-

ence on the distribution of velocities, because the adjustment of velocities which counteracts this consequence of the circulating movement is also a slow process on account of internal friction.

We have now revealed the causes of the formation of meanders. Certain details can, however, also be deduced without difficulty from these facts. Erosion will inevitably be comparatively extensive not

FIG. 3.

merely on the right-hand wall but also on the right half of the bottom, so that there will be a tendency to assume the shape illustrated in fig. 3.

Moreover, the water at the surface will come from the left-hand wall, and will therefore, on the left-hand side especially, be moving less rapidly than the water rather lower down. It should further be observed that the circular motion possesses inertia. The circulation will therefore only achieve its maximum extent behind the position of the greatest curvature, and the same naturally applies to the asymmetry of the erosion. Hence in the course of the erosion an advance of the wave-line of the meander-formation is bound to take place in the direction of the cur-

rent. Finally, the longer the cross section of the river, the more slowly will the circular movement be absorbed by friction; the wave-line of the meander-formation will therefore increase with the cross section of the river.

Lecture 19 Porous media Fluid Flow

I Hydrologic Cycle

Every year about 1 m evaporates from each m^2 of the earth's surface. Plants are so good at evapotranspiration that the evaporation rate on land is similar to over water (except in very arid areas). Since $A_E \sim 5.1 \times 10^{14}\ m^2$, $\sim 510{,}000\ km^3$ of water evaporates each year. About 10% of the evaporation over the oceans ($\sim 3/4 \times 510{,}000\ km^3 \times 0.1 = 38{,}000\ km^3/yr$) moves over and precipitates on the continents and then eventually (overland & underground) returns to the oceans.

VG #2, 4, 5 from 201 Hydrology

Rain forest Implication

But the continental rainfall is (this is the ocean part) $\sim 110{,}000\ km^3/yr$. "Thinking" of the rain that falls, 1/3 is from ocean and 2/3 from local evaporation.

The large rivers (Amazon #1 $5510\ km^3/yr$, ..., Mississippi #7 $551\ km^3/yr$) account for about 37% of the runoff.

II. Subsurface fluid flow

Flow in the subsurface is driven by surface topography, volume change + compaction (squeeze sponge), and buoyancy (convection).

All are important for different reasons

- Topography — hydrology, stream flow
- volume change / compaction — hydrocarbons, foundations
- buoyancy — ore deposits (gold)

We will briefly discuss all of these — starting with hydrology

Where to start — Henry Darcy.

- ~~Compaction~~ is due to stress contact in permeability = just function

cementation, structure, ang

A. Darcy's law

The conservation of momentum equation that describes the flow of water through porous media was determined, elegantly, by a set of experiments in 1856 by Henry Darcy and published in an appendix to a ground water supply report on Municipal water supply he prepared for the city council. The easiest way to obtain Darcy's law is from the Navier-Stokes equation — simply or in more detail:

1. Simple extraction of Darcy's Law

(19-1) Navier Stokes

$$\rho \frac{D\underline{v}}{Dt} - \rho g + \nabla p - \mu \nabla^2 \underline{v} = 0$$

- If no turbulence $\rho \frac{D\underline{v}}{Dt} = 0$

- World to fluid is one pore. If pore diameter is d

$$\nabla^2 v \rightarrow -\frac{\mathcal{S}}{d^2}\,\underline{v}$$

Minus sign because gradient of flow decreases outward; $\mathcal{S}$ just arbitrary constant.

- Define Darcy Flux $\underline{V} = \underline{v}\phi$, where $\underline{v}$ is the average fluid velocity in the open cracks of the sediment or rock. The area or volume fraction of these cracks or pores is ϕ.

With these changes the Navier-Stokes equation becomes

$$V = -\frac{d^2\phi}{\mathcal{S}\mu}\left(\underline{\nabla} p - \rho g\right)$$

(19-2) Darcy's Law

$$\boxed{V = -\frac{k}{\mu}\left(\underline{\nabla} p - \rho g\right)}$$

huge range!

where $k \equiv$ intrinsic permeability of the porous medium with dimensions of m^2.

Hydrologists specialize this equation

assuming gravity and fluid density are constant. So lets

simplify like they do. We can write:

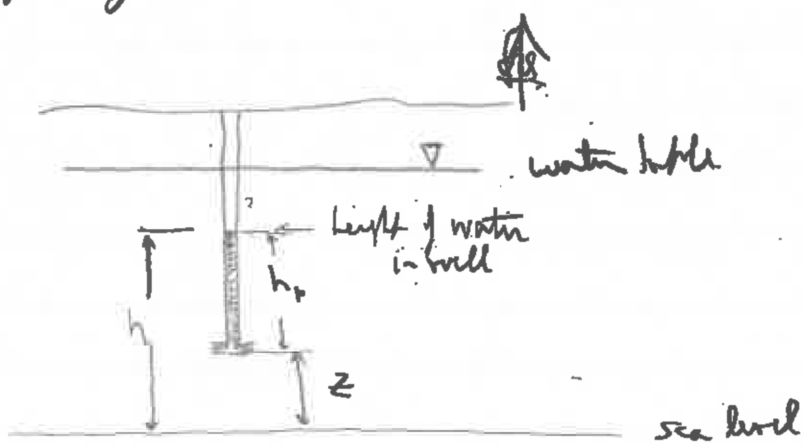
$$\underline{V} = -\frac{k}{\mu} \nabla (p + \rho g z)$$

where z is measured positive up. We can define elevation

head, z; pressure head h_p, and hydraulic head, h, relative

to the depth of perforations in a well:

sketch



Bucket test – Huge Range!

$$p \equiv (h - z) \rho_0 g$$

$$V = -\frac{k}{\mu} \nabla \left(h \rho_0 g - \cancel{z \rho_0 g} + \cancel{\rho_0 g z} \right)$$

(19-2a) Darcy's Law

$$\boxed{V = \frac{-k \rho_0 g}{\mu} \nabla h = -K \nabla h}$$

$\left\{ \begin{array}{l} \nabla h = 1 \\ V = K \end{array} \right.$ (just set)

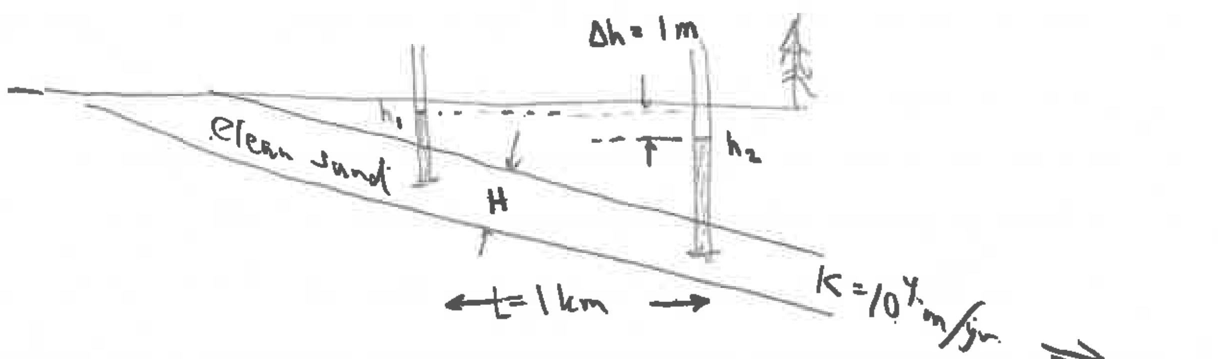
where K is the hydraulic conductivity in m/sec.

This is the form of the conservation of momentum equation discovered experimentally by Henry Darcy. Notice hydraulic conductivity, K combines intrinsic permeability, water density, earth gravity, and water viscosity. Darcy's law is specialized to water.

It's beauty is that it is extremely easy to apply.

For example suppose the water level in two wells penetrating a sandstone layer have water levels as indicated below:

sketch



1m/yr

$$V = -K\left(\frac{h_2 - h_1}{L}\right) = 10 \text{ m/yr}$$

If the thickness of the aquifer was 10 m, the total flux through the aquifer per m width would be

$$Q = V \cdot H = 100 \text{ m}^3/\text{m-yr}$$

B. Geometric (continuity) constraint on fluid flow:

Suppose the subsurface was of uniform permeability. What would subsurface flow look like then?

$$V = -k \nabla h$$

continuity $\nabla \cdot V = 0 \Rightarrow \nabla^2 h = 0$

$$h = X(x)\, Z(z)$$

$$Z \frac{\partial^2 X}{\partial x^2} + X \frac{\partial^2 Z}{\partial z^2} = 0$$

$$\frac{1}{X} \frac{\partial^2 X}{\partial x^2} + \frac{1}{Z} \frac{\partial^2 Z}{\partial z^2} = 0$$

$$\begin{array}{cc} \shortparallel & \shortparallel \\ -k & +k \end{array}$$

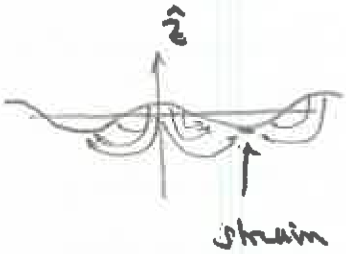
$X = A \cos kx + B \cancel{\sin kx} \rightarrow 0$ — want the cos kx function

$Z = C \cancel{e^{-kz}} + D e^{+kz} \rightarrow 0$ — finite solution

∴ Solution is

(-4)

$$\boxed{\begin{aligned} h &= h_0 \cos kx \; e^{kz} \\ V_z &= -K \frac{\partial h}{\partial z} = -K h_0 k \cos kx \; e^{kz} \\ V_x &= -K \frac{\partial h}{\partial x} = K h_0 k \sin kx \; e^{kz} \end{aligned}}$$



(19-5)

$$\boxed{\frac{V_z}{V_z(z=0)} = e^{kz}}$$

$z_{50\%\ \text{flow}} = 0.1\,\lambda$ ⟸ shallow

$$\frac{\partial h(z=0)}{\partial t} = \frac{V_z(z=0)}{\phi} = \frac{-Khk}{\phi}$$

$$\int_{h_0}^{h} \frac{\partial h}{h} = \int_0^t \frac{-Kk}{\phi}\, dt$$

(19-6)

$$\boxed{h = h_0 e^{-t/\tau}, \quad \tau = \frac{\phi}{Kk}}$$

Just like glacial rebound!

Streams continue flowing because the mound of the water table under hills take a long time to decay.

Example If there were a drought, how long would the streams run? If $I = 0$ the water table will fall as $h = h_0 e^{-t/\tau}$.

Since stream recharge is proportional to h, the stream discharge will fall exponentially in the same fashion. The streams will discharge at ~1/3 their starting rate $\left(= \frac{1}{e \cong 2.7}\right)$ when $t = \tau$.

$$\tau_{\text{This case}} = \frac{\phi}{kK} = \frac{.1}{(6.28 \times 10^{-4}\ m^{-1})(1.5\ m/yr)}$$

$$\tau = 106 \text{ years}$$

This is a bit long for Ithaca, which suggests

The permeable channels that feed our creeks are, at least in part, considerably more permeable. Perhaps 100 md rather than 1 md.

Early

Our analysis suggests that in single (e.g. monsoonal) saturation river discharge should decay exponentially. This is in fact observed. For example:

DETERMINING GROUND-WATER RECHARGE FROM BASEFLOW

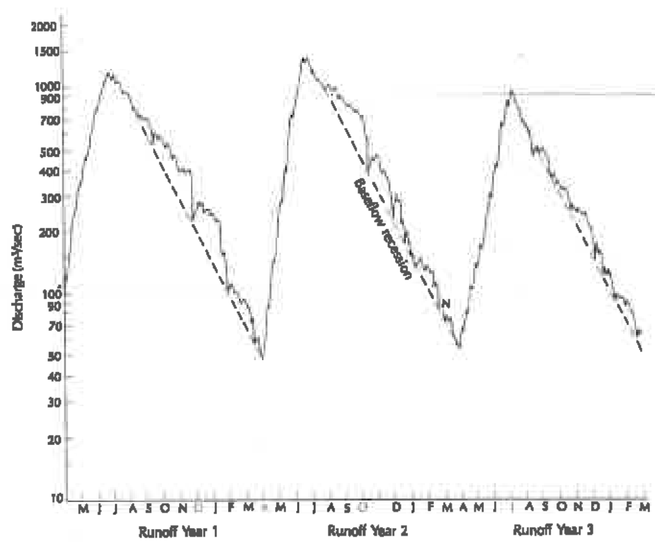


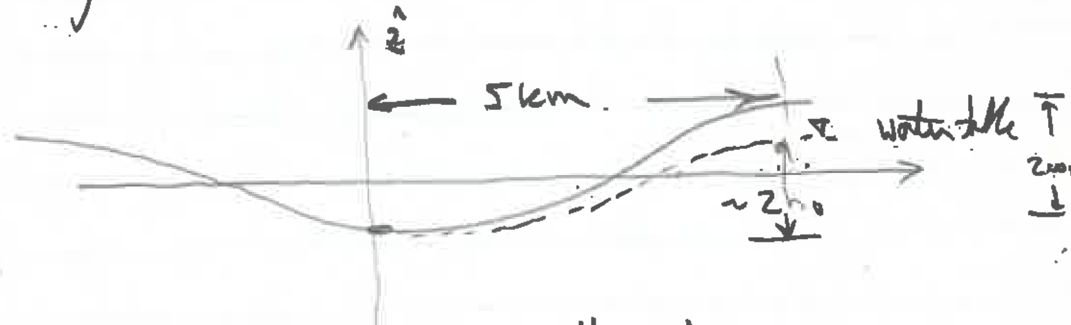
FIGURE 3.15 Semilogarithmic stream hydrographs showing baseflow recessions.

V_g #4

Exercise: Assume $\lambda = 10$ km, $\phi = 10\%$ find the permeability of this area. What lithology would you expect?

continue We can use our knowledge of rainfall infiltration and the height of the water table under hills to determine large scale permeability.

<u>For example</u> suppose the hills crests are ~5 km from the stream (as in Ithaca). The <u>height of the water table</u> under the hills will be determined by the infiltration rate and the permeability



$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{10\,\text{km}} = 6.28 \times 10^{-4}\ \text{m}^{-1}$$

$$I = 0.1\ \text{m/yr} = 3.17 \times 10^{-9}\ \text{m/s}$$

$$h_0 = 100\ \text{m}$$

$$V_z(z=0) = I = -K h_0 k$$

$$K = \frac{I}{h_0 k} = \frac{3.17 \times 10^{-9}\ \text{m/s}}{(100\ \text{m})(6.28 \times 10^{-4}\ \text{m}^{-1})} = 5 \times 10^{-8}\ \text{m/s} = 1.5\ \text{m/yr} \cong 1\ \text{millidarcy}$$

— quite a typical permeability!

C. Convective flow

$$q = \rho V = \frac{-k}{\nu}\left(\nabla p + \rho g \hat{z}\right)$$

(19-7) No flow condition

$$q = 0 \Rightarrow \boxed{\frac{\partial p_0}{\partial z} = -\rho_0 g}$$ No Flow

Let

$$\rho = \rho_0(z) + \rho_1(x, y, z, t)$$

$$p = p_0(z) + p_1(x, y, z, t)$$

Substituting

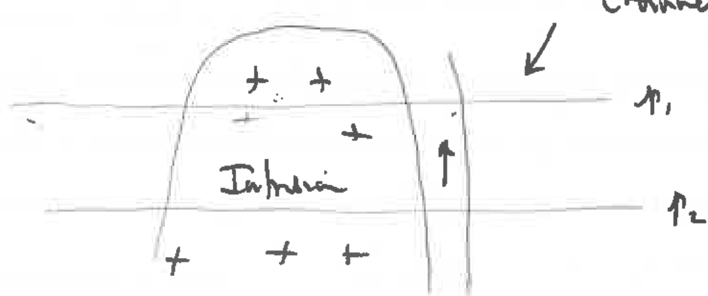
$$q = \frac{-k}{\nu}\left(\cancel{\nabla p_0} + \nabla p_1 + \cancel{\rho_0 g \hat{z}} + \rho_1 g \hat{z}\right)$$

(19-8) Darcy's Law

$$\boxed{q = \frac{-k'}{\nu}\left(\nabla p_1 + \rho_1 g \hat{z}\right)}$$ Flow

where p_1 is excess-hydrostatic pressure (equivalent to h)

channel collapses until $p_1 = 0$



350°C
flow zone

Then

in channel 0

$$q = -\frac{k'}{\nu}\left(\nabla p_1 + \rho_1 g \hat{z}\right)$$

$$\boxed{q = -\frac{k}{\nu}\rho_1 g \hat{z} = \frac{k}{\nu}\left(\alpha \rho_{00} \Delta T g\right)\hat{z}}$$

$\alpha = 10^{-3}\ {}^\circ K^{-1}$

$\Delta T = 250\ {}^\circ C$

$g = 10\ m/sec^2$

$\nu = 2\times10^{-7}\ m^2/sec$

$k = 350\ md = 350\times10^{-12}\ m^2$

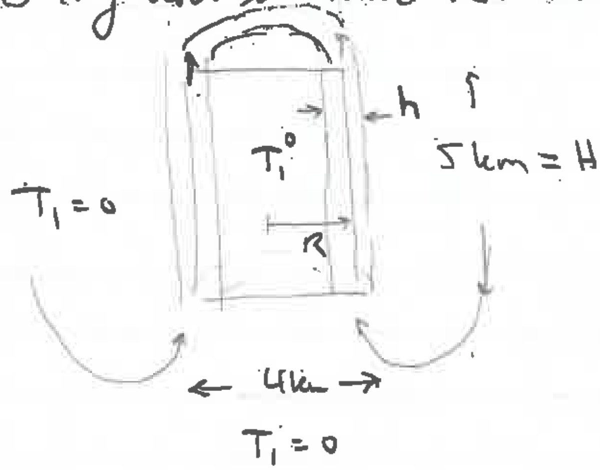
darcy?

Suggested for mid ocean ridge flow zone (black smokers)

$q = 4.37\times10^{-3}\ kg/m^2\text{-sec} = 1.37\times10^{5}\ kg/m^2\text{-yr}$

$V = \frac{q}{\rho} \cong 137\ m/yr$

How long does it take an intrusion to cool?



Heat contained by intrusion $= T_r^0 \rho_m \bar{C}_m \pi R^2 H$

Rate of heat removal $= 2\pi R q C h \Delta T$

$$T_{cool} = \frac{T_1 \rho_m C_m \cancel{\pi} R^{\cancel{2}} H}{2\cancel{\pi}\cancel{R}\, q C h \Delta T}$$

(-9)

$$\boxed{T_{cool} = \left(\frac{T_1}{\Delta T}\right)\left(\frac{\rho_m C_m}{\rho C}\right)\left(\frac{R}{h}\right)\frac{H}{V}}$$

$$\frac{T_1 = 800^\circ C}{\Delta T = 250^\circ C} = 3.2$$

$$\frac{\rho_m C_m}{\rho C} \cong 0.5$$

$$\frac{R}{h} = \frac{2000}{3.4} = 588$$

suggested by mid-ocean ridge analysis

$$\frac{H}{V} = \frac{5000\,m}{137\,m/yr} = 36\,yrs$$

compatible with 10 yr residence time of seawater > 150 °C

$$\underline{\underline{T_{cool} = 33,000\,yrs}}$$

Conductive cooling of a cylinder, side set to 0°C at $t = 0$

cool when $\frac{\kappa t}{R^2} \cong 0.4$

$$t = 50,000\,yrs$$

Convective cooling requires a fairly permeable host (for water inflow) ~ 1 millidarcy. Some intrusions cool by conduction and have metamorphic halos; some cool convectively and have no halos but possibly form are deposits. The cutoff of host permeability for vigorous convection cooling is quite sharp. A host permeability of < 0.1 md will cause intrusion to cool conductively. A host permeability of 1 to 10 md will guarantee vigorous convective ~~cooling~~

D. Overpressuring

$$\frac{1}{V}\frac{DV}{Dt} = -\alpha \frac{D\sigma_{eff}}{Dt}$$

total volume

$$\alpha \equiv \frac{-1}{V_T}\frac{DV_T}{D\sigma_{eff}}$$

$$\sigma_{eff} = \sigma_T - p$$

If $\sigma_T = \text{const}$, water withdrawal causes $\downarrow p$ and $\uparrow \sigma_{eff}$

$$\alpha = \frac{-1}{V_T}\frac{\partial V_T}{\partial \sigma_{eff}} = \frac{-1}{H}\frac{\partial H}{\partial \sigma_{eff}} = \frac{1}{H}\frac{\partial H}{\partial p}$$

$$= \frac{1}{H}\frac{\partial H}{\partial \rho g h}$$

height of water table

$$\partial H = H \alpha \rho g \, \partial h$$

$\alpha = 10^{-6}\ Pa^{-1}$ clay

10^{-7} to $10^{-9}\ Pa^{-1}$ sand

$$\underbrace{\rho}_{1000}\ \underbrace{g}_{10}\ \underbrace{\alpha}_{<10^{-6}} \cong .01$$

$$\partial H = .01\, H\, \delta h$$

for $H = 100$ m

$\partial H \sim \delta h$ $\leftarrow$ change in elevation = change in water table !

I E. Permeability details

The issue with hydrology is the extremely wide range of permeability in the earth's crust. If ground is just saturated, $\nabla h = 1$ and $V = K$. Thus K can be thought of as the rate at which water will drain into the ground. (septic tank test):

Vg#1

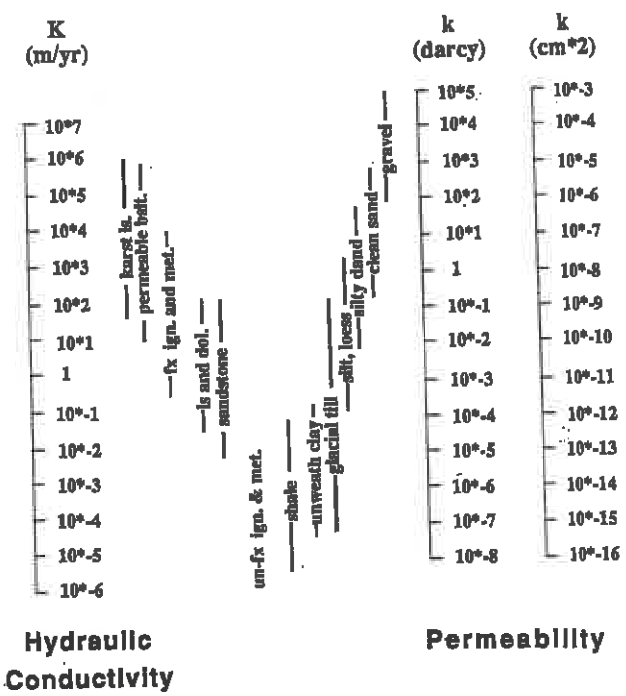


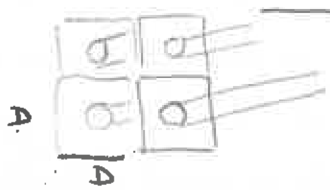
FIGURE 5.2 Hydraulic conductivities and permeabilities of common sediment and rock types (adapted from Freeze and Cherry, 1979).

Water can drain into ground at 100's of km/yr or at < 1 μm

To understand subsurface flow we must understand the connections of high permeability units. This is one of main jobs of geology and it is quite challenging (styrofoam cup with pinhole problem).

2. Permeability from Pipe flow

Kundu + Cohen p 285



Consider a porous media (sandstone) as a set of pipes. The flux that each pipe is:

$$Q = \frac{\pi a^4}{8\mu} \frac{dp}{dx}$$

since $\phi = \frac{\pi a^2}{D^2}$ and $\frac{Q}{D^2} = V$,

$$V = \frac{\phi a^2}{8\mu} \frac{dp}{dx} = \underbrace{\left(\frac{a}{2}\right)^2 \frac{\phi}{2}}_{=k} \frac{dp}{dx}$$

now $$\frac{a}{2} = \frac{\text{Vol pore}}{\text{Surface area of pore}} = \frac{\pi a^2 L}{2\pi a L} = \frac{\phi}{(1-\phi)\, S_{vp}}$$

where S_{vp} = surface area of solid / volume of the solid. For a sand

$S_{vp} =$  $= \frac{4\pi r^2}{(2r)^3} = \frac{\pi}{d}$

(19)

Thus for sand $k = \left(\frac{a}{2}\right)^2 \frac{\phi}{2} = \left(\frac{\phi}{1-\phi}\frac{d}{\pi}\right)^2 \frac{\phi}{2}$, or

$$k = \frac{\phi^3}{(1-\phi)^2}\frac{d^2}{2\pi^2} \qquad , \left(2\pi^2 = 19.7\right) \times 3^2 = 178$$

$$d_{throat} = \frac{d_{grain}}{3}$$

If the pore the pore throats are ~1/3 the diameter of the grains

(19-3) Karmen Kozeny relation

$$\boxed{k = \frac{\phi^3}{(1-\phi)^2}\frac{d^2}{180}}$$

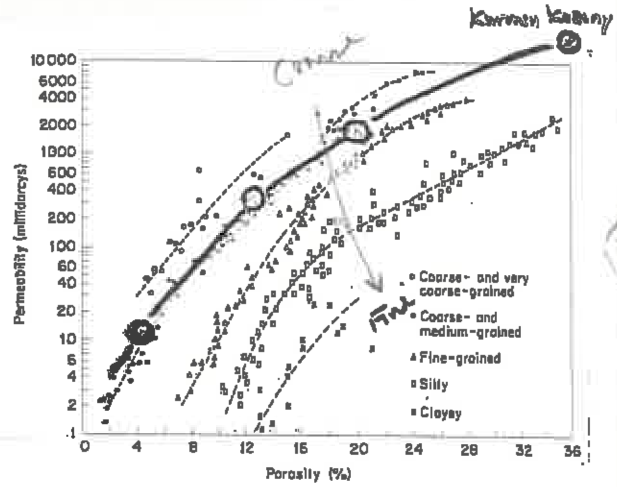


Figure 4.5 Relationship between porosity and permeability for sandstone in various grain-size categories (after Chilinger, 1963).

This Karmen Kozeny relation does very well in fitting the permeability of sands and sandstones

1 darcy = $10^{-8}\ cm^2 = 10^{-14}\ m^2$. In general permeability is strongly dependent on porosity and porosity decreases with depth due to compaction

- Note importance of pore throats (constrictions).

- Geophysical tools to measure permeability are holy grail of geophysics. Electrical conductivity is very promising because it is very sensitive to pore throats (where water conductivity is greatly enhanced!)

20

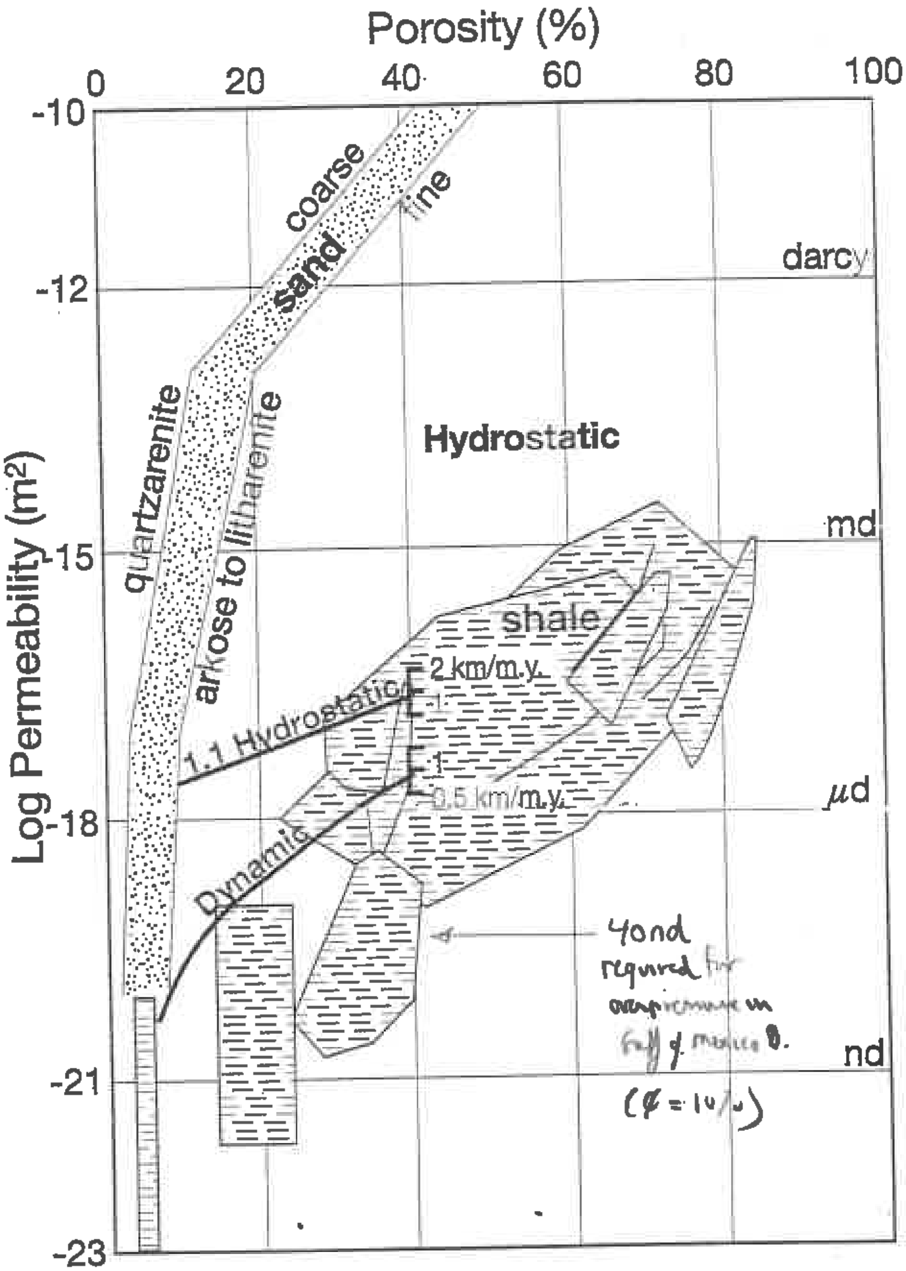


FIG. 17. Logarithm of permeability plotted against porosity for sandstones (Nelson, 1994; Nelson, 2000) and shales (Neuzil, 1994). Coarse quartz arenites are the most permeable lithology. The curve labeled "Dynamic" represents the minimum permeability that shales with different porosities (depths of burial) can have when the sedimentation rate is 1 km/m.y. The ticks at the right end of the curve indicate how the curve will shift if the sedimentation rate varies between 0.5 and 2 km/m.y. The curve marked "1.1 Hydrostatic" represents the permeability that basin sediments must have for pore pressures to exceed hydrostatic by only 10 percent at sedimentation rates between 0.5 to 2 km/m.y.

VF#3

F. Compaction Details

Consider that the fluid density ρ_1 = constant = 0

Then

$$q = -\frac{k}{\nu} \nabla p_1$$

where p_1 is the excess hydrostatic pressure. For a material volume conservation of fluid and solid mass lead to the following mass conservation equation:

(19-10) Fluid Conservation $$\frac{D(\rho\phi)}{Dt} + \nabla \cdot q + \phi\rho\left(\frac{1}{V}\frac{DV}{Dt}\right) = 0 .$$

fluid

Here V is the total volume of the material parcel (solid + fluid

$$\frac{D(\rho\phi)}{Dt} + \nabla \cdot q + \phi\rho\left(\frac{1}{V}\frac{DV}{Dt}\right)$$

$$\parallel$$

$$\rho\frac{D\phi}{Dt} + \phi\rho\frac{1}{V}\frac{DV}{Dt} + \nabla \cdot q = 0$$

$$\parallel$$

$$\rho(1-\phi)\frac{1}{V}\frac{DV}{Dt}$$

$$\frac{1}{V}\frac{DV}{Dt} + \frac{1}{\rho}\nabla \cdot q = 0$$

0 for incompressible ($\rho_s = 0$) fluid

$$\frac{D(\rho\phi)}{Dt} = \phi \cancel{\frac{D\rho}{Dt}} + \rho \frac{D\phi}{Dt}$$

Solid mass conservation

$$\frac{D\phi}{Dt} = (1-\phi)\frac{1}{V}\frac{DV}{Dt}$$

So equation (10) becomes (see overleaf)

(19-10a)
$$\boxed{\frac{1}{V}\frac{DV}{Dt} + \frac{1}{\rho}\nabla\cdot q = 0}$$

The media volume is changed by effective stress

$$\frac{1}{V}\frac{DV}{Dt} = -\alpha\frac{D\sigma_{eff}}{Dt}$$

(19-11)
$$\alpha \equiv -\frac{1}{V}\frac{DV}{D\sigma_{eff}}, \quad \sigma_{eff} = \sigma_T - p.$$

Thus (10a) becomes:

$$-\alpha\frac{D\sigma_{eff}}{Dt} + \frac{1}{\rho}\nabla\cdot q = 0$$

(19-10b)
$$\boxed{\alpha\left(\frac{Dp}{Dt} - \frac{D\sigma_T}{Dt}\right) - \frac{1}{\rho}\nabla\cdot\frac{k}{\mu}\nabla p_1 = \phi}$$

$$\alpha\left(\frac{Dp}{Dt} - \frac{D\sigma_T}{Dt}\right) - \frac{1}{\rho}\nabla\cdot\frac{k}{\mu}\nabla p_1 = 0$$

$$\alpha\left(\cancel{\frac{Dp_0}{Dt}} + \frac{Dp_1}{Dt} - (\rho_s - \rho_w)(1-\phi_0)\,g\,S - \cancel{\frac{Dp_0}{Dt}}\right) - \frac{1}{\rho}\nabla\cdot\frac{k}{\mu}\nabla p_1 = 0$$

σ_T is the total (vertical) stress on the surface and can be changed by increasing water depth in a lake or ocean or adding sediments

$$\frac{D\sigma_T}{Dt} = \underbrace{(\rho_G - \rho_{00})}_{\text{Density of mineral grains}}\overbrace{(1-\varphi_0)}^{\text{depositional porosity}}\, g\, \underbrace{S}_{\text{sedimentation rate}} + \underbrace{\frac{Dp_0}{Dt}}_{\text{change in water depth}}$$

deposited in water

$$\frac{Dp}{Dt} = \frac{Dp_0}{Dt} + \frac{Dp_1}{Dt}$$

Thus

(19-10c)
$$\boxed{\alpha\left(\frac{Dp_1}{Dt} - (\rho_G-\rho_{00})(1-\varphi_0)\, g\, S\right) - \frac{1}{\rho}\nabla\cdot\frac{k}{\nu}\nabla p_1 = 0}$$

If $k \to 0$, the excess hydrostatic pressure increases by the buoyant weight of sediments deposited. If the permeability is such that the water can drain at the rate the sediment load is applied, $p_1 = 0$ (e.g. there is no overpressure.

sp storage ← more [illegible] for unit head

Thickness ← more sedimentation

$$100 = \frac{S_s\, l^2}{K t} = \frac{S_s\, l\, \omega}{K}$$

sedimentation rate

$$t = l/\omega$$

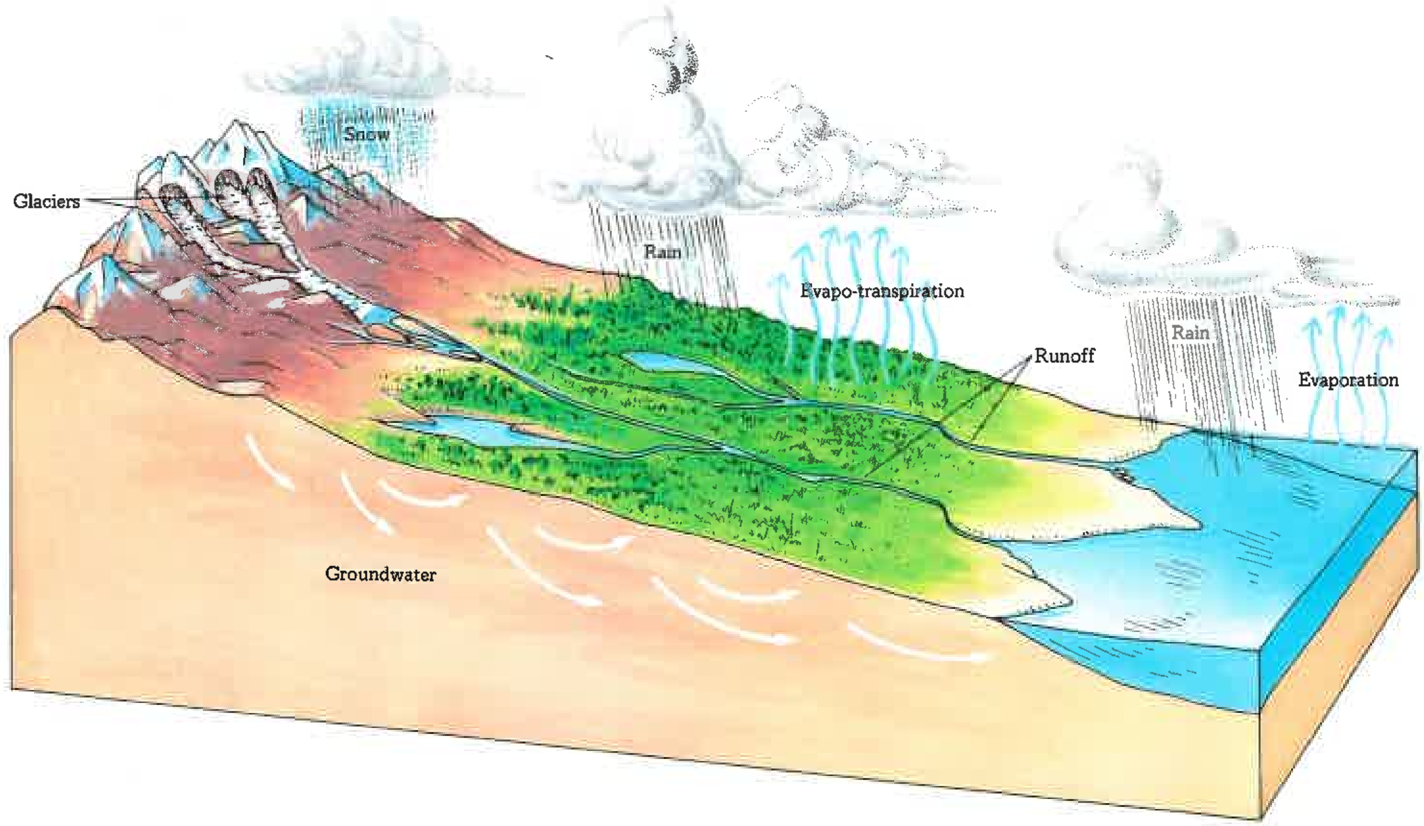
hyd cond

more difficult to expel water

$$\frac{\Delta V}{V} \Big/ \Delta t = S_s$$

greater is porosity more sed can store more fluid for given change in head

HYDROLOGIC CYCLE
(Fig. 13.2, p. 243)

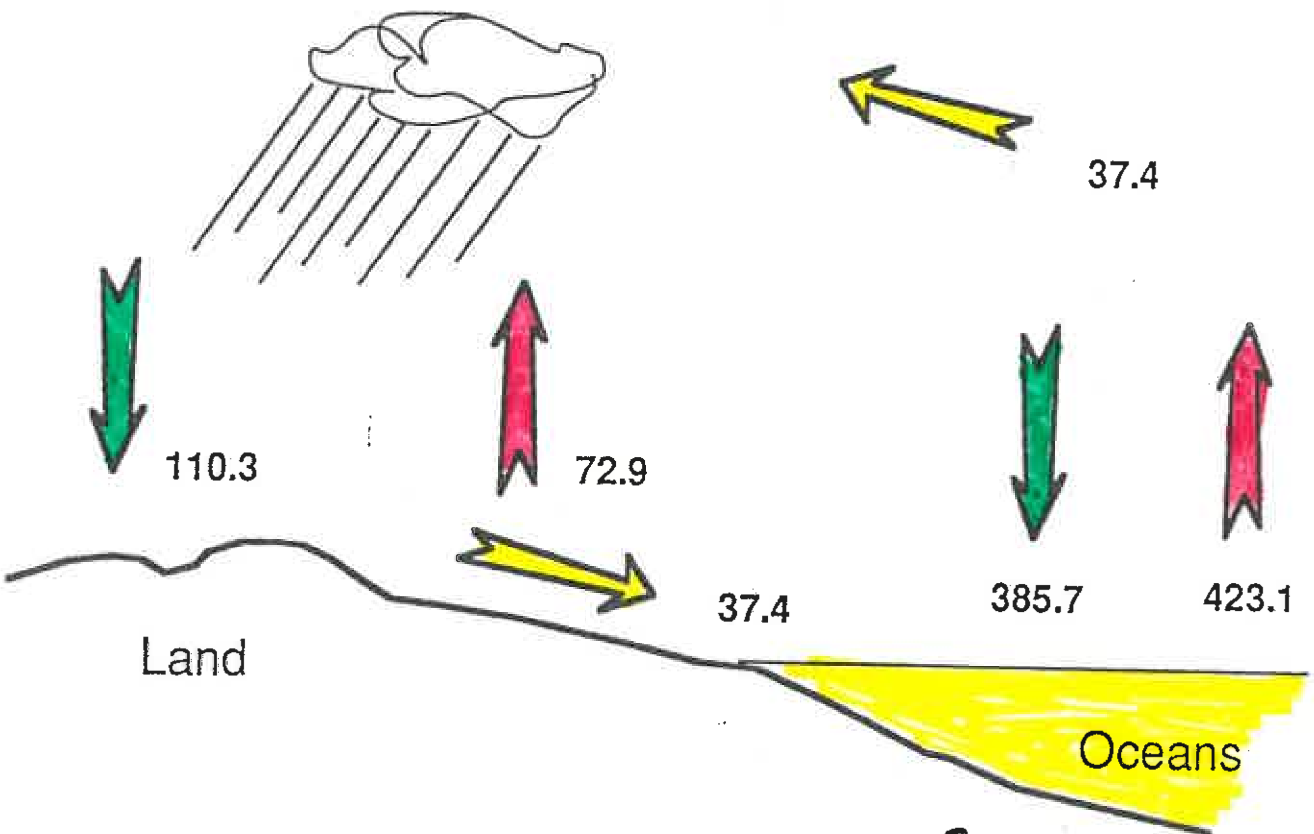


PHYSICAL GEOLOGY, 7th ed.
by Judson, Kauffman, and Leet

2

A

Evap = 496 km³/yr



Fluxes in the Hydrologic cycle (in 10^3 km3 / yr)

10^3 km³/yr

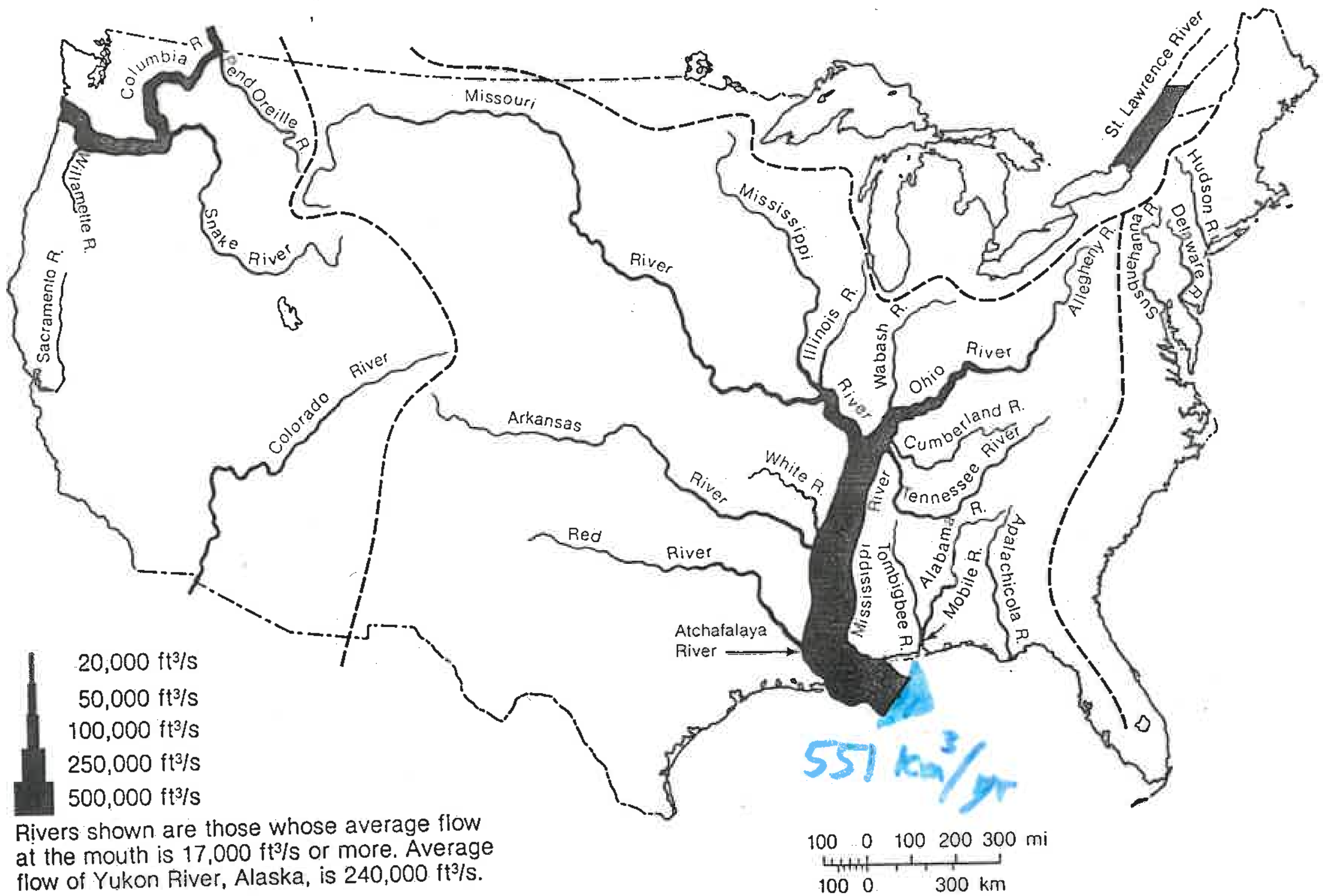
RIVER DISCHARGE

Amazon, S.Am	5510 km^3/yr
La Plata, S.Am	2500
Congo, Af	1250
Yangtse, China	687
Bramaputra, Asia	624
Ganges, Asia	589
Mississippi, N.Am	551

11700 km^3/yr
(37% Total Runoff)

TOTAL RIVER DISCHARGE = 37,400 km^3/yr

THE MAJOR U.S. RIVER IS THE MISSISSIPPI



Rivers shown are those whose average flow at the mouth is 17,000 ft³/s or more. Average flow of Yukon River, Alaska, is 240,000 ft³/s.

Continental divides

5

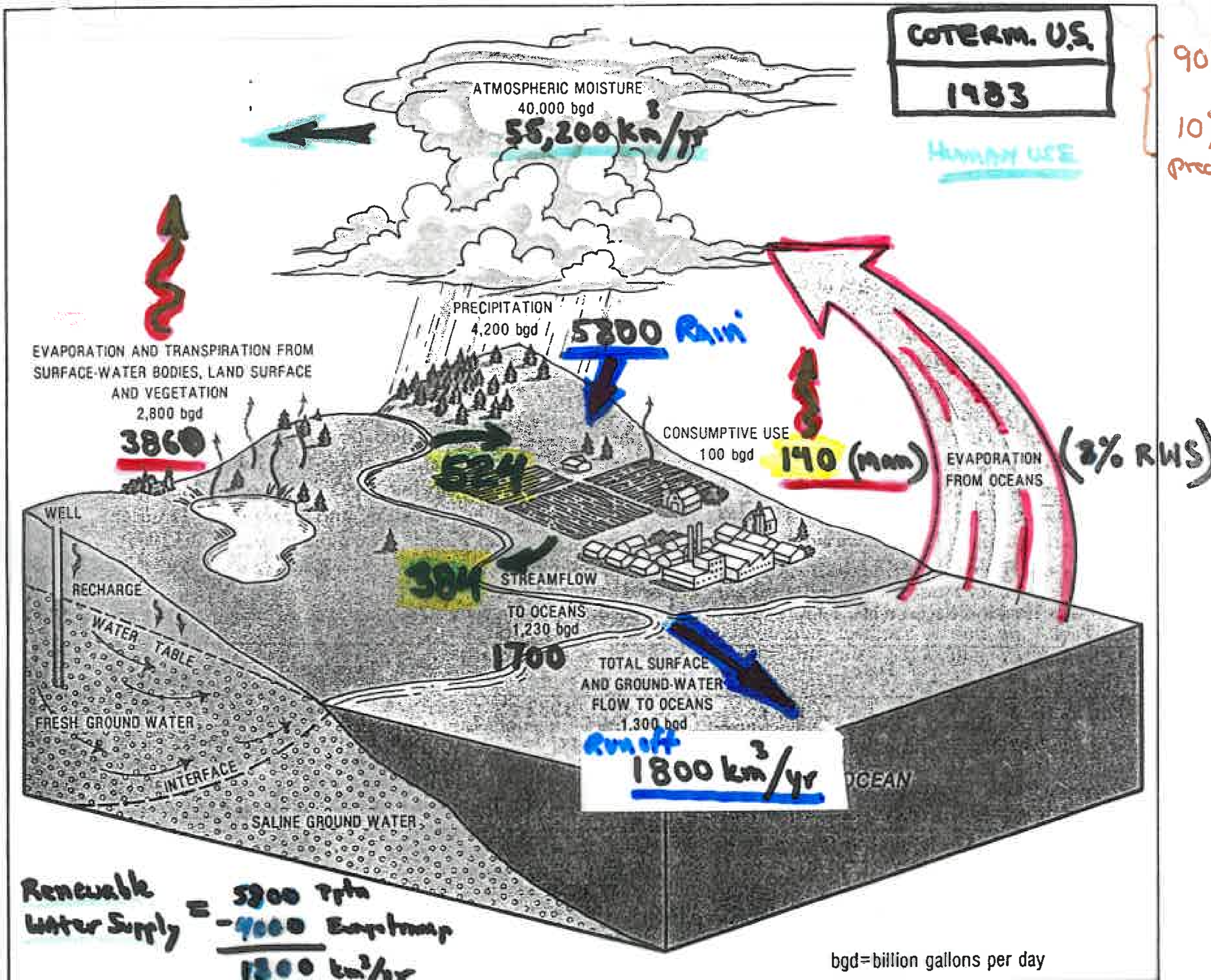
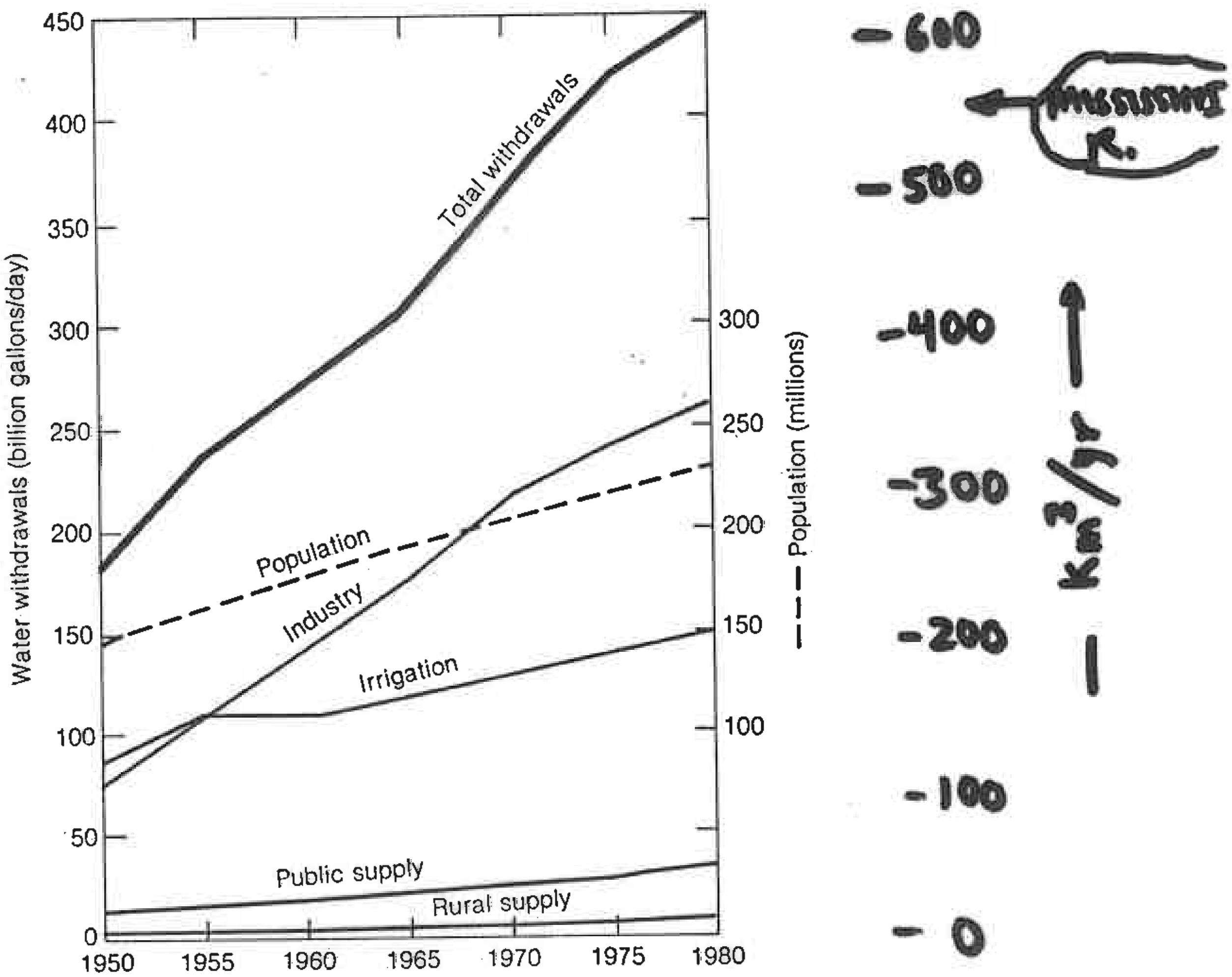
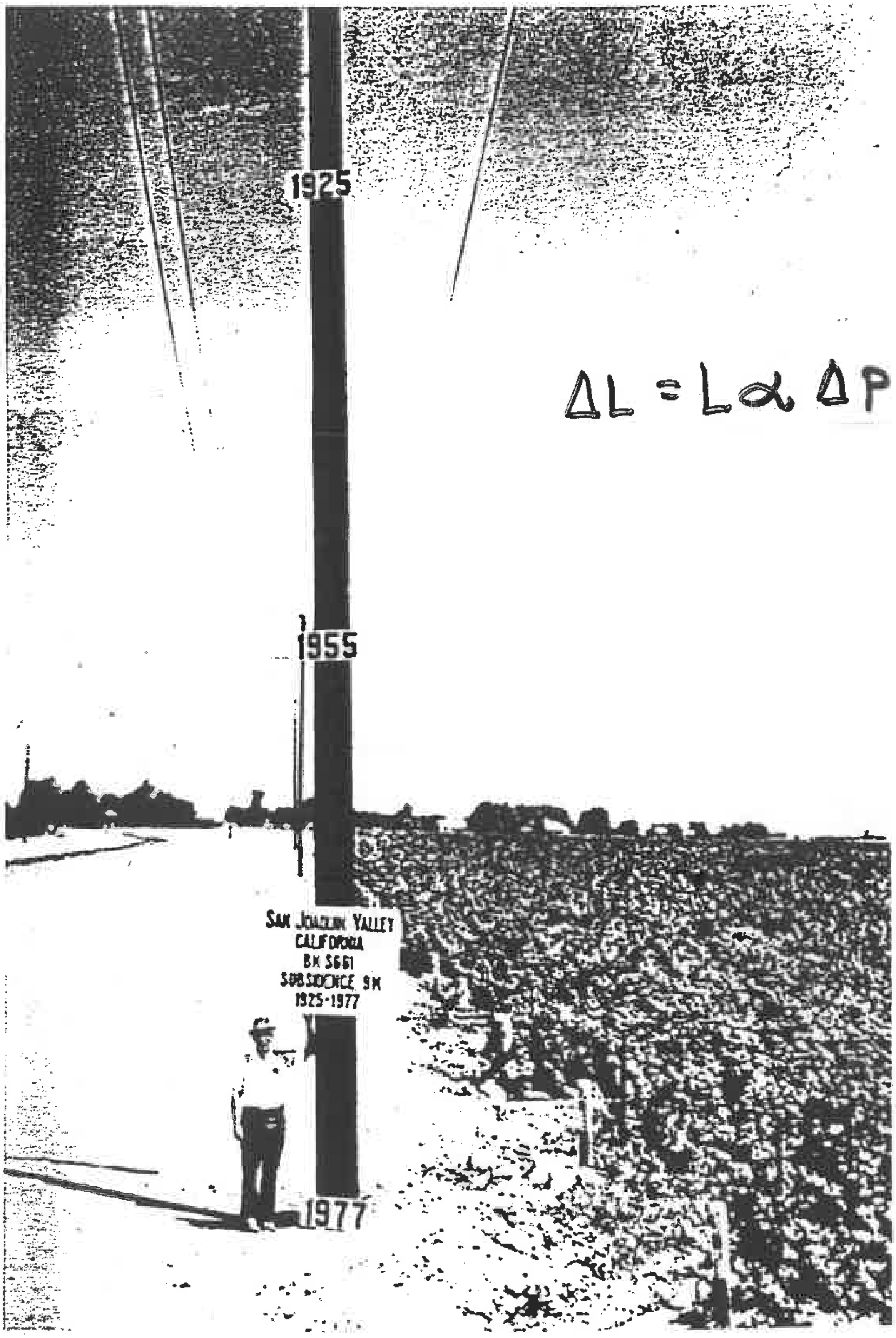


FIGURE 1. Hydrologic cycle showing the gross water budget of the conterminous United States. (Data from Langbein and

WATER WITHDRAWALS ARE INCREASING





$$\Delta L = L \alpha \Delta P$$

Dr. Joseph F. Poland stands at the approximate point of maximum subsidence in the San Joaquin Valley, California. Subsidence of approximately 9.0 m occurred from 1925 to 1977 due to aquifer compaction caused by pumping of ground water. Signs indicate the former elevations of the land surface in 1925 and 1955 respectively. Photo taken December 1976.

sp storage ← more sorption for unit load

Thickness ← more residence time

$$100 = \frac{S_s \ell^2}{K t} = \frac{S_s \ell \omega}{K}$$

$t = \ell / \omega$

sedimentation rate

hyd cond'y

↑

more difficult to expel water

$$\frac{\Delta V}{V} \Big/ \Delta t = S_s$$

greater is lowerly mean sed can store more fluid for given change in head

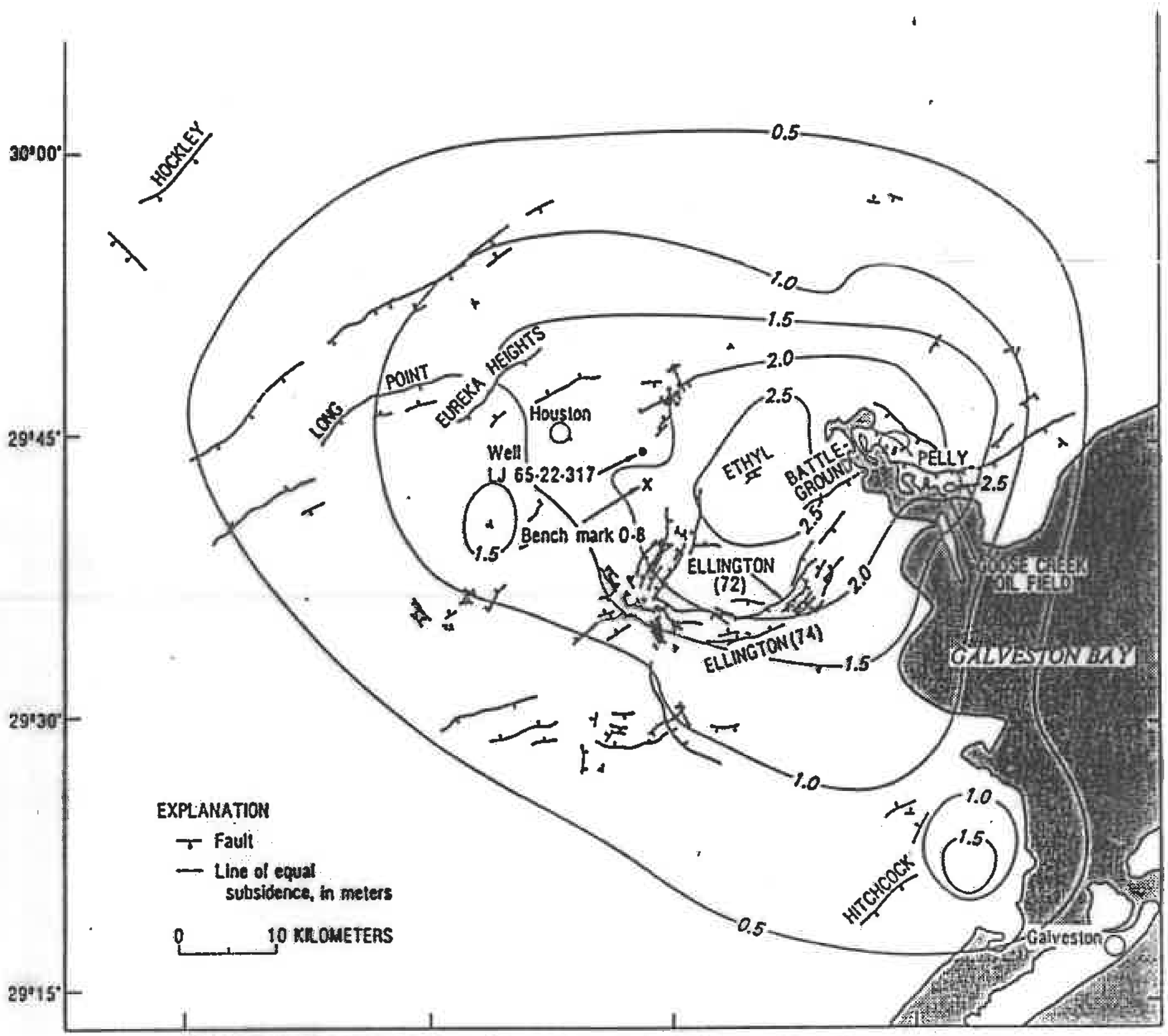


Figure 13. Land subsidence from 1906 to 1978 (adapted from Gabrysch, 1980) and surface faults (Elsbury and others, 1980; Verbeek and Clanton, 1981; E. R. Verbeek, 1981, unpub. data) in the Houston-Galveston, Texas, region. Ball-and-bar symbols on faults indicate downthrown side. Named faults are referred to in text or figures.

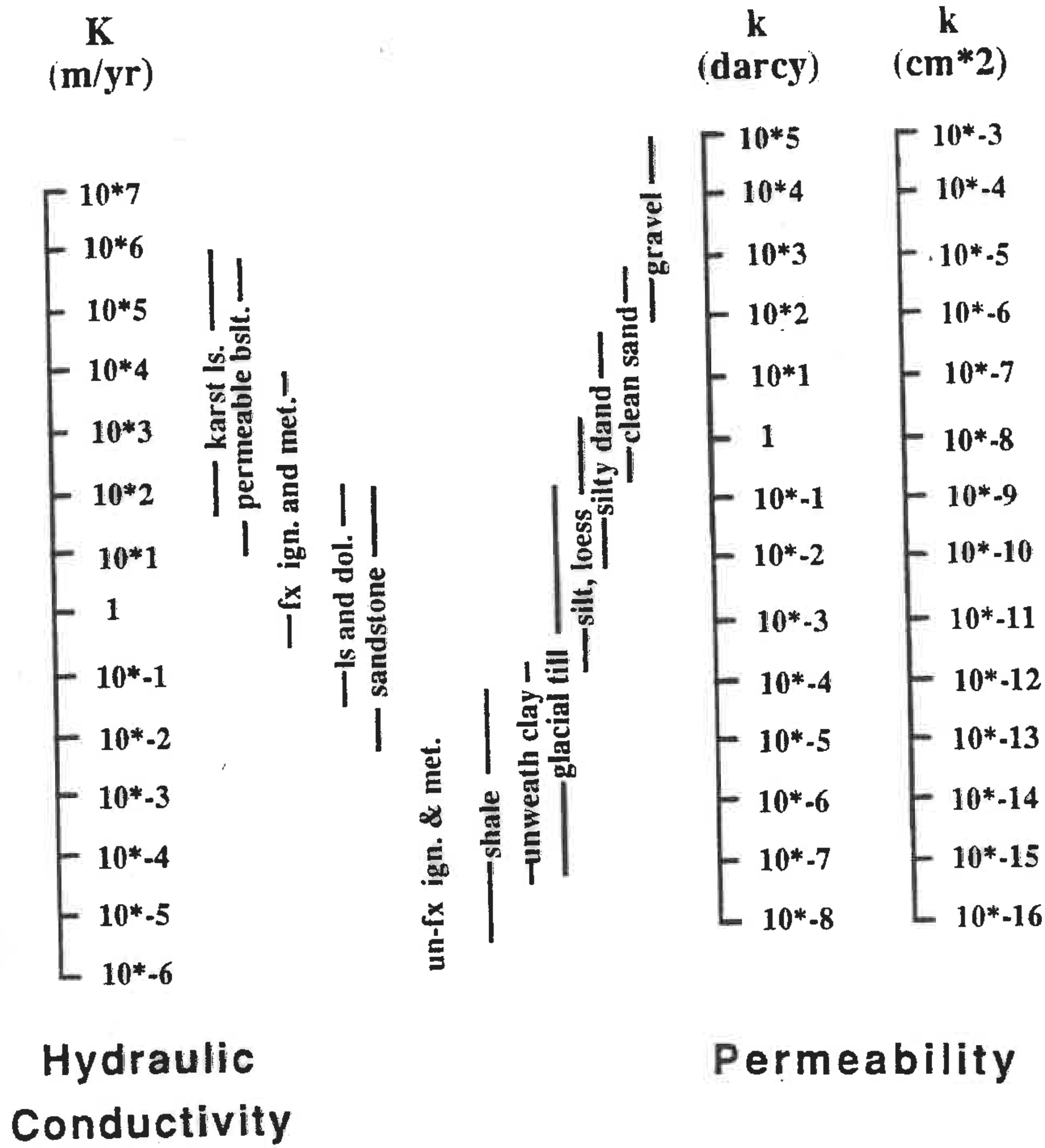


FIGURE 5.2 Hydraulic conductivities and permeabilities of common sediment and rock types (adapted from Freeze and Cherry, 1979).