APPENDIX IV The Importance of Capillary Forces in Smearing out the Flow from Waste Dump Ponds (by P. J. Ring)

This appendix shows capillary forces can be validly neglected so far as their effect on the flow from waste dump ponds is concerned. Flow from dump ponds will be essentially vertical, capillary smearing being about one foot for a typical dump.

These predictions have apparently been confirmed by recent excavation of a Cananea dump where water was found to move "straight down" from leach ponds*.

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* John Apps conveyed this information from a recent meeting with Anaconda dump experts.

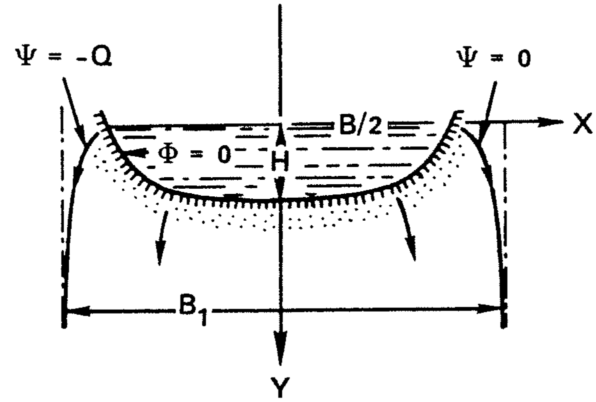


Figure 1. Ditch profile of gravity flow problem discussed by Muskat. $\psi = -Q$ and $\psi = 0$ are the streamlines corresponding to the free surfaces of the flow sheet.

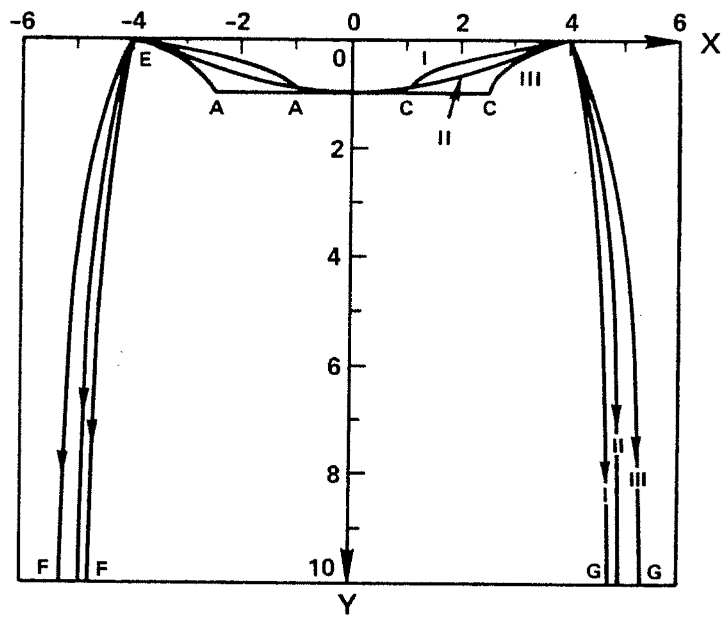


Figure 2. Free surface streamlines for several ditch profiles. All profiles have the same width and depth. Profile II is the same as that shown in Figure 1.

The Influence of Capillarity on Gravity Flow (Flow from a shallow pond)

Two phase flow through a porous media may be affected by capillary forces. The physical origin of these forces and the criteria for evaluating their significance are discussed in the following paragraphs.

A pressure differential is produced at the boundry of two fluids by the surface tension (surface energy density) at their interface, (Morrow, 1970). This pressure difference is called the capillary pressure, P_c, and is a function of the mean radius of the surface. The capillary pressure is defined as the difference between the pressure in the fluid that lies on the concave side of the interface and the pressure in the other fluid.

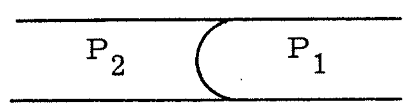


Fig. 1.

For the pressures P_1 and P_2 of Figure 1 the capillary pressure P_c is

$$P_c = P_1 - P_2.$$

For a cylindrical tube containing water, where the water wets the solid (tube) surface, P_c is given by

$$P_c = 2\sigma / R,$$

where σ is the surface tension (surface energy density) of the water and R the radius of the spherical surface.

If the interfacial surface is not spherical the capillary pressure can be expressed in the form

$$P_c = \sigma / R_m,$$

where R_m is a constant, sometimes called the mean radius.

$$\frac{1}{R_m} = \frac{1}{R_1} + \frac{1}{R_2}.$$

R_1 and R_2 are the radii of curvature of any two orthogonal cross sections of the interface, (Morel-Seytoux, 1969). Thus, it is always possible to describe the capillary pressure in terms of a single parameter R_m, though this value need not be the "spherical" radius of the interface or the diameter of the capillary "cylinder". This radius is not necessarily a pore diameter for the porous media either.

In porous media the capillary parameter R_m will not be constant but will have a distribution of values analogous to the pore size distribution. Measurements of the static capillary pressure as a function of water saturation in air-water systems show the affect of this pore size distribution quite clearly. Figure 2 shows the results for measurements on sandstone cores. The larger values of capillary pressure correspond to the distribution of pores with small diameters. It is thus these smaller diameter pores (6 microns and less) that retain the 30% water saturation, that is the connate water of the core. For pores having diameters of 10 microns and larger the water should be relatively free to move. It is noted that the capillary pressure, P_c, vs. water saturation, S, has three regions of different slopes. Two of these have relatively large slopes and one a relatively small slope. The regions of large slope are near saturation S = .9 to 1.0 and below S = .3. The significance of the slope of the P_c, vs. S curve will be discussed in later paragraphs.

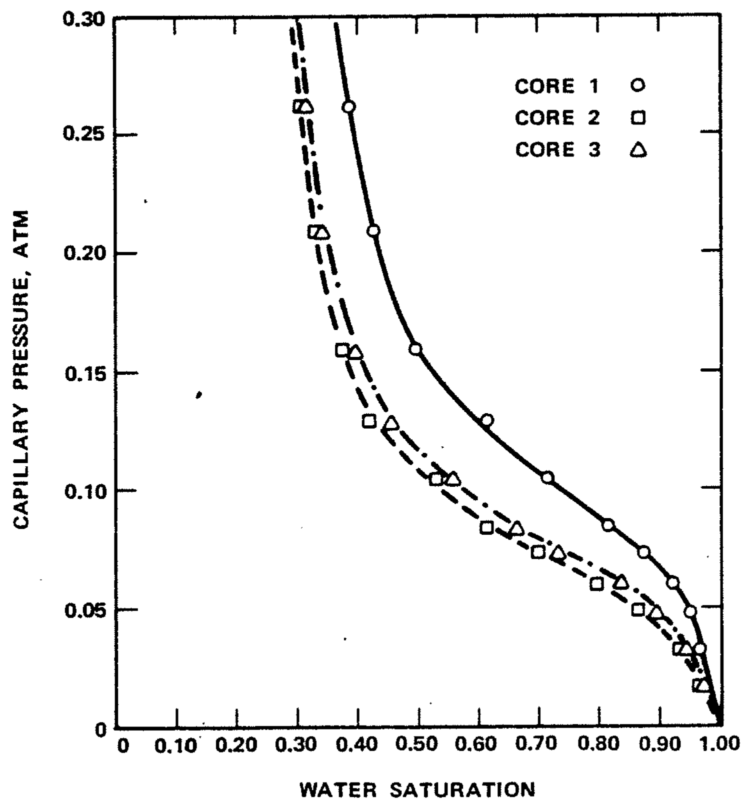


Fig. 2. - Static capillary pressure as a function of saturation for sandstone cores (air-water system, drainage curve). (Handy, 1960).

Figure 3 shows the capillary pressure vs. saturation for both drainage and imbibition. The hysterisis of the curves is apparent. Both drainage and imbibition have curves of similar shape. It should be noted that these are curves for the static capillary pressure.

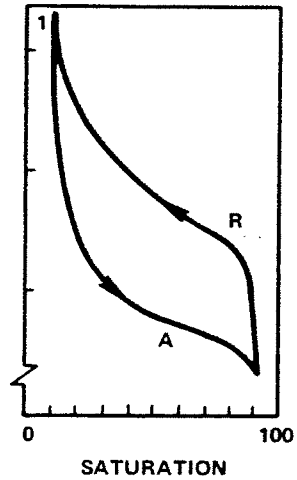


Fig. 3. - General behavior of capillary pressure scanning curves. R, drainage; A, imbibition.

From the preceding paragraphs it would appear that both porosity and permeability should depend on saturation. Permeability, k, the measure of ease of fluid flow is defined as

$$k = -\frac{Q}{A} \quad \frac{\mu}{\rho g} \quad \left(\frac{\partial h}{\partial s}\right)$$

where

(Q/A)	=	the volume of fluid crossing an area A in unit time
μ	=	the viscosity of the fluid
ρ	=	the density of the fluid
g	=	the acceleration of gravity
$\partial h/\partial s$	=	the hydraulic gradient in the direction of the flow.

The porosity is not strictly speaking saturation dependent. However, the actual pore volume available for fluid flow will depend on fluid pressure, since the fluid pressure must exceed the capillary pressure for flow to occur through a region of given capillary radius. A fraction of the available pore volume will not be available except at pressures much greater than one atmosphere and thus must almost always be excluded from the pore volume when evaluating the porosity of the media.

From a consideration of the fluid flow in a single capillary one might expect that the permeability would be independent of saturation for driving pressures above a threshold pressure determined by the geometry of the tube and the surface energies of the fluid and solid. For a porous media there is a continuous range of geometries and thus of driving pressures. This driving pressure depends on the capillary pressure which in turn depends on the saturation. Thus the permeability will be a function of the saturation of the porous media. Figure 4 shows the dependence of the relative permeabilities of gas and water in unconsolidated sands (Wyckoff and Botset, 1936), on the saturation.

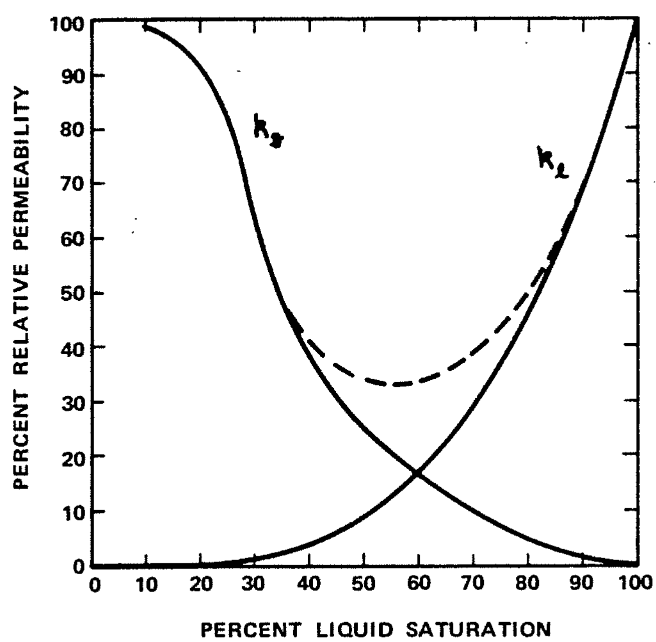


Fig. 4. - Permeability-saturation data for four sets of unconsolidated sands. k_l, k_g refer to liquid and gas phases. The dashed curve represents the sum of the gas and liquid permeabilities. (Wyckoff and Botset, Physics, 1936).

Four characteristics of the relative permeabilities of Figure 4 should be noted: 1) The rapid decrease of the liquid permeability with small changes of saturation in the region of 100% saturation, 2) The zero value of the gas permeability for values of saturation greater than 90%, 3) The approach of the liquid saturation to zero at fairly large values of liquid saturation (30%) and 4) The approach of the gas permeability to 1 for saturation much greater than zero (10 to 20%). Physically this behavior may be interpreted as follows. For high saturations, 90 - 100%, the gas fills the larger voids and is trapped there. The blocking of these regions, though not significant in displacing water, does reduce the flow in those small number of larger diameter channels that are responsible for a significant volume of the water flow and thus there is a significant reduction in the water permeability. At the lower saturations the water becomes essentially trapped in the smaller diameter pores and thus its permeability becomes essentially zero. This occurs at the region of the connate water saturation due to the large capillary forces. The gas flow is then no longer effected by the presence of the second phase except that the effective porosity is somewhat less than for single phase flow. Thus below the connate water saturation the gas permeability is approximately unity, (Muskat, 1946).

Flow in the porous media is obtained from the Darcy Law written for two immiscible fluids: that is from the equations (Morel-Seytoux, 1969),

$$V_w = -k \left[k_{rw} (S_w) / \mu_w\right] \text{ GRAD } \left[P_w + \rho_w gZ\right]$$

$$V_a = -k \left[k_{ra} (S_w) / \mu_a\right] \text{ GRAD } \left[P_a + \rho_a gZ\right]$$

where

V_w	is the velocity of the water
V_a	is the velocity of the air
k	is the intrinsic permeability of the media
$k_w(S'_w)$ and $k_a(S_w)$	are the relative permeabilities of the water and air respectively. These are functions of the water saturations S_w.
μ_w and μ_a	are the viscosities of the water and air respectively.
P_w and P_a	are the pressures in the water and air.
ρ_w and ρ_a	are the densities of the water and air.
g	is the gravitational constant
z	is the vertical coordinate with up as the positive direction.

When capillary forces are present these equations can be written as, (Muskat, 1949)

$$V_w = -k\left[k_{rw}(S_w)/\mu_w\right] \text{GRAD}\left[\bar{P} - \frac{1}{2}P_c + \rho_w gZ\right]$$

$$V_a = -k\left[k_{ra}(S_w)/\mu_a\right] \text{GRAD}\left(\bar{P} + \frac{1}{2}P_c + \rho_a gZ\right)$$

where

$$\bar{P} = (P_w + P_a)/2.$$

These later equations show that the capillary effects enter as the gradient of the capillary pressure. This gradient can be written, by means of the chain rule, as

$$\frac{\partial P_c}{\partial s} = \left(\frac{\partial P_c}{\partial S}\right)\left(\frac{\partial S}{\partial s}\right).$$

Thus the capillary contribution will be significant when the product of the two partial derivatives become significant compared to grad $\bar{P}$. The capillary forces will be negligible in the interior of a flow sheet where the gradient of the saturation is very small ($\partial S/\partial s = 0$). At the free surface of the flow sheet one can expect capillary forces to be more important because the gradients of saturation are large. The level of saturation is low implying a large value of $\partial P_c / \partial S$.

As an example of the effects of capillarity on flow one can consider radial flow into a well, a problem discussed by Muskat (1946). A diagram of the configuration is given in Figure 5.

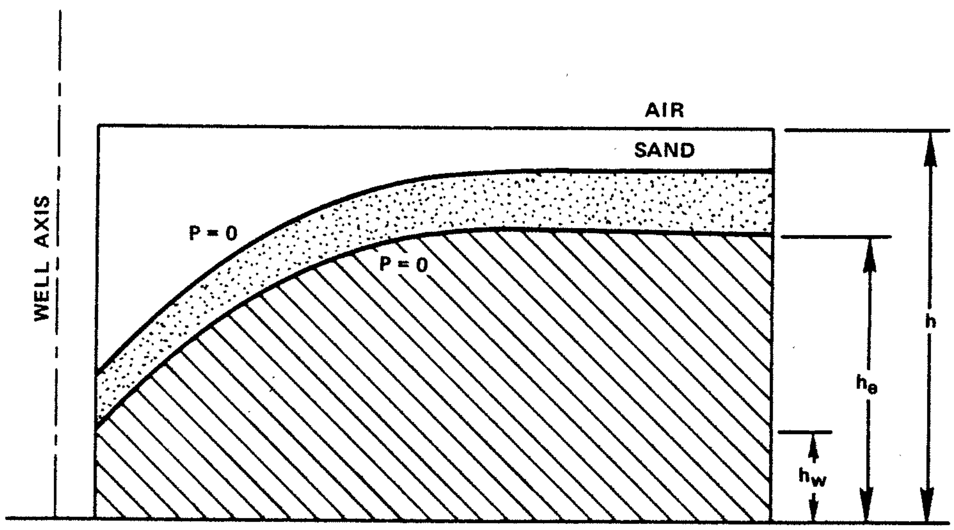


Fig. 5. - Diagrammatic representation of the effect of the capillary layer, h_c, on a gravity-flow system. h_e is the reservoir head, h_w the well head and h the well depth.

Water from the reservoir having head h_e flows both through the saturated region (lined section) and through the capillary regions (dotted section). Muskat shows that when the fluid height is such that the capillary layer above this height cannot attain its maximum thickness before being terminated at the air-sand interface, then the flow in this incompletely developed capillary layer may be neglected. (The maximum height of the capillary layer is determined by the pore size of the sand). If the capillary layer is well developed it acts as a flow channel and can contribute significantly to the total flow. Results of his calculations and our calculations from a "Rule of Thumb" which will be developed subsequently, and a comparison of these estimates to observed flows are given in Table I. The value of the sand

Run No.	h_e (cm)	h_w (cm)	Calc. Q (cc./sec.) (Without Capillary)	Calc. Q (cc./sec.) (With Capillary)	Calc. Q (Rule of Thumb)	Obs. Q.
1	23.67	18.07	3.06	3.76		3.90
2	15.37	0.17	3.09	4.98	5.02	4.95
3	14.37	0.17	2.70	4.47	4.51	4.55
4	9.57	0.17	1.18	2.37	2.37	2.50

h_c = 9.5 cm. h = 29 cm.

Table I. Comparison of Observed and Calculated Fluxes Through a Radial Gravity-Flow System

height, h, was kept at 29.0 cm., and the pore size was such that the capillary height, h_c, was 9.5 cm.

When the capillary layer is not completely formed the term $(\partial S/\partial s)$ will be very large, particularly at the surface. This will result in such large capillary forces that no fluid can flow. When the capillary layer is formed the influence of the capillary forces within the capillary layer diminish and flow can exist. That is, $(\partial S/\partial s)$ becomes smaller and the value of S is such that $(\partial P_c/\partial S)$ is also small, resulting in a smaller contribution of the capillary pressures to the flow. The consequence is that the capillary regions now become zones in which there is fluid flow and the total volume flow increases. As seen in run number one of Table I, the contribution of the capillary zone is small when the relative size of the capillary zone is small. If one assumes that for a fully developed capillary layer the affect of that layer is to increase the cross section available for flow and that the actual flux is the same as for the porous media without capillary forces, development of a "rule of thumb" becomes possible. In Muskat's problem the augmentation of cross-sectional area available to flow is approximately:

$$\frac{h_e + h_c}{h_e}$$

We choose the cross section far away from the well where near well distortions should be negligible. Results of augmenting the normal flow by the above factor are given in the sixth column of Table I. The agreement of the observed flow, the "rule of thumb", and with capillary flow calculations suggests the validity of the rule of thumb.

Risenkamph as quoted by Polubarinova - Kochina (1962) has discussed the flow from canals with low (zero) water levels. The results of his analysis are shown by the curves in Figure 6. In that Figure B is the physical width of the canal (pond), Q_o the total flow for the pound without capillary forces, Q the total flow rate of the pound when capillary forces are considered, B_1 the width of the wetted zone, B_2 the width of the seepage flow at

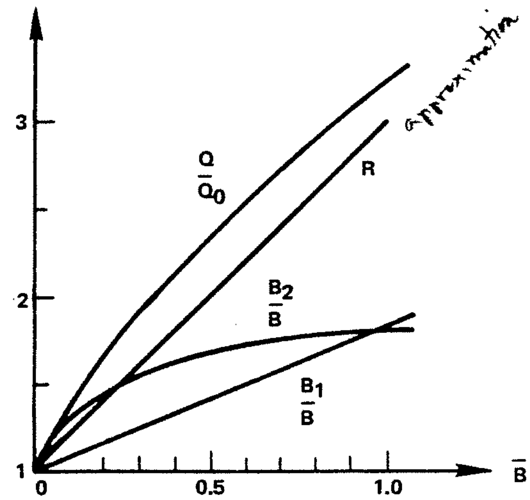


Figure 6. Flow from canals with zero water level

infinity. If h_c is the capillary head, the increased area available to flow because of capillarity is:

$$R = 1 + 2 (h_c/B).$$

Our rule of thumb suggests the increased flow can be estimated by increasing the radius of the pond by the length of the capillary head. As can be seen from a comparison of curves for Q/Q_o and R in Figure 6, this approximation appears valid to about 15% of the actual flow. (It is always less than the actual flow).

Physical origins of the spreading of the streamlines of a pond (dump) are demonstrated in Figure 7. In this figure the ratio h_c/B is assumed to be 0.5, and the parameter B_1/B is obtained from Figure 6. The parameter B is the pond dimension and B_1 the dimension of the wetted region. Flow without capillary affects would occur within the cross hatched region of Figure 7a only (assuming a pond of zero depth). If $\nabla P_c = 0$ (Fig. 7b), the flow is vertical and the resistive force, $F\mu$ exactly balance the gravitational force producing the flow $g\overline{z}$. Capillarity of the porous media results in the wetted region, $(S > 0)$ increasing in dimension from B to B_1. This occurs because ∇P_c is no longer zero near the edge of the saturated region. As shown in Figure 7c and 1 and 2 of Figure 7a ∇P_c has the effect of enlarging the saturated zone by producing a horizontal component of the flow velocity.

The flow does not extend indefinitely in the horizontal direction since $\partial S/\partial s$ eventually becomes quite small. Recall that ∇P_c is composed of two parts

$$\nabla P_c = \left(\frac{\partial P_c}{\partial S}\right)\left(\frac{\partial S}{\partial s}\right).$$

The first term of the chain is always negative and is maximum in regions of high and low saturation. The saturation becomes small as one moves away from the free surface and thus its slope is negative. Thus the gradient of P_c is positive and the force it produces is away from the boundary defined by B_1 and consequently produces the siphon like effect in the capillary region. Since the saturation approaches a constant value the term $\partial S/\partial s$ becomes very small and the horizontal component of the capillary force becomes zero. This causes the capillary envelope of the flow zone to have finite dimensions in all practical cases.

From the foregoing discussion it is evident that the relevance of capillary forces to flow in porous media is directly related to the pore size (capillary radius) distribution in the media and the dimension of the wetted region of fluid flow. For the two flow problems examined it was shown that an estimate of the capillary contribution to the total flow could be obtained from the ratio of the capillary pressure head to the fluid flow dimension in the absence of capillary forces, (i.e. the dimensions of a dump). Figure 8

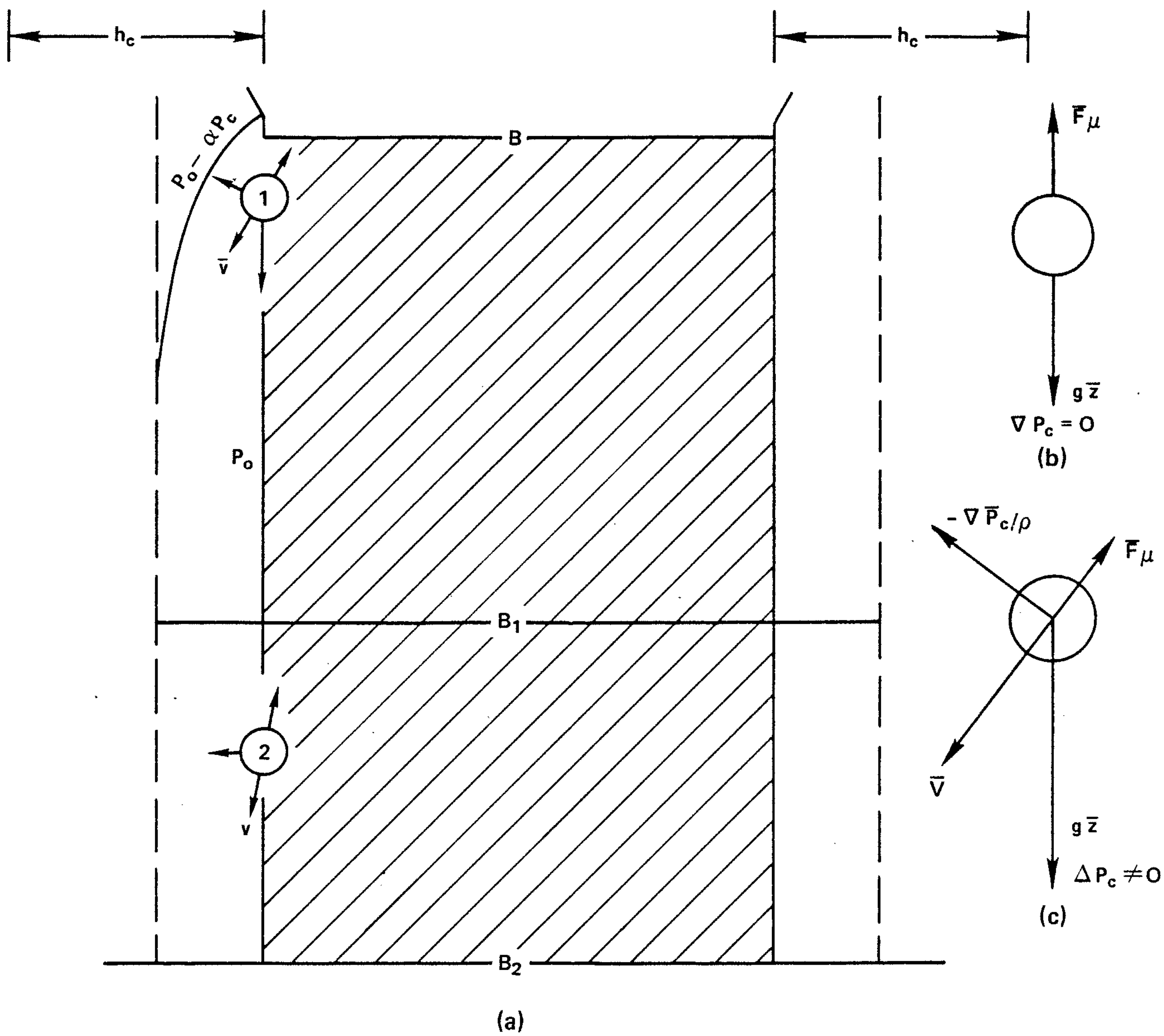


Figure 7. Physical Origins of the Spreading of the Streamlines of a Dump (Pond). When there is a gradient in the water saturation ∇P_c is no longer zero. Capillary forces enlarge the volume of the dump through which flow can take place.

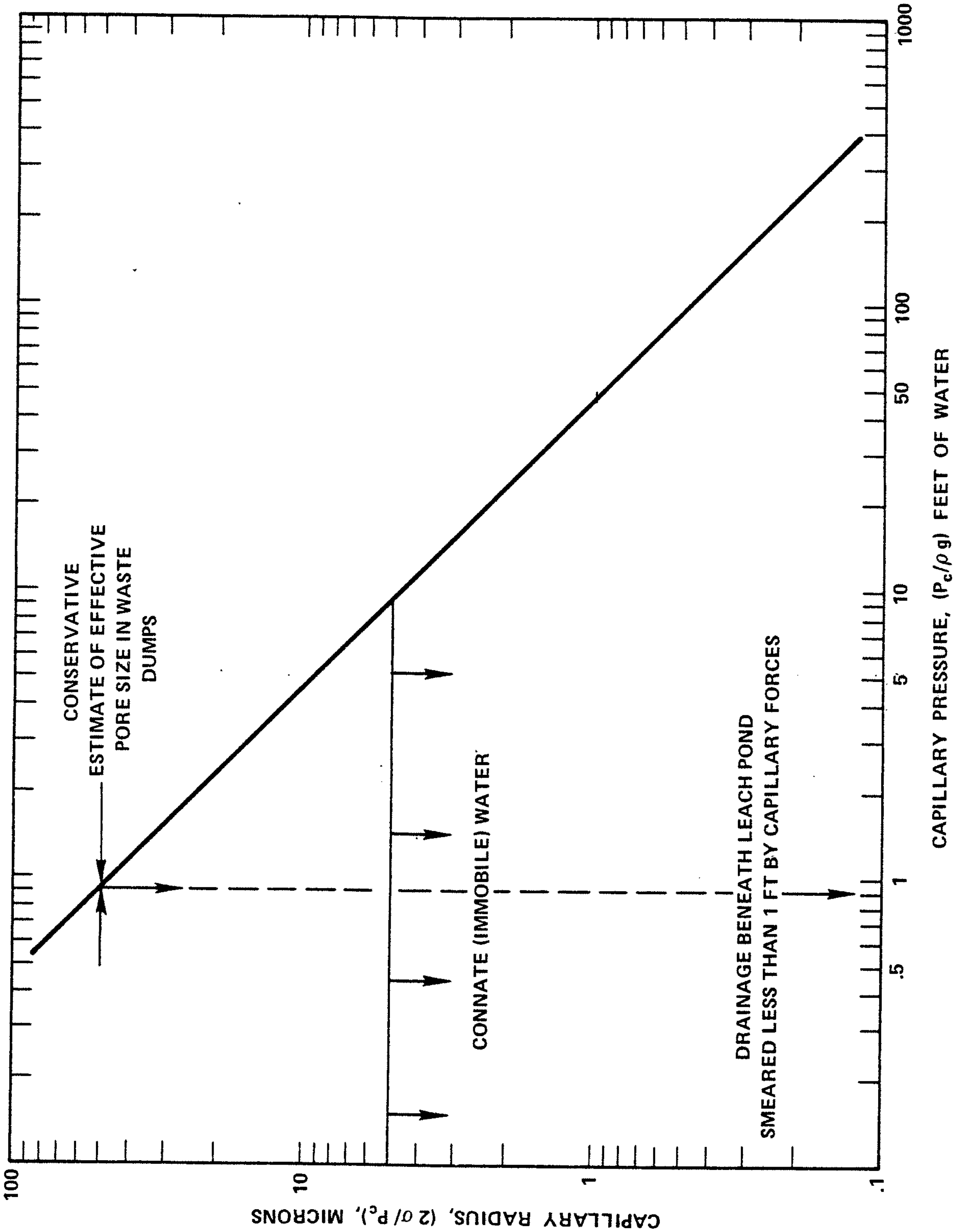


Figure 8. Capillary pressure versus capillary radius.

gives a curve showing the relationship between capillary pressure head and capillary radius. It is to be noted that capillary pressures are below atmospheric pressures for radii greater than about 1.4 microns.

If one assumes that a porous medium can be characterized by some effective pore size, even though it contains a distribution of capillary (pore) sizes, then that effective pore size can be equated with the hydraulic head and thus the influence of capillarity on the flow. Assuming that an increase of 10% or less is a negligible contribution to the total flow, the curve of Figure 9 has been drawn relating pond dimension to effective pore size. Thus the solid line corresponds to a capillary height of 1/10th of the indicate pond size and the effective pore size corresponds to the capillary size for that capillary height as obtained from Figure 8.

The fact that the distribution of pore sizes may contain a number of pores of sizes much smaller than the effective pore size will not alter the basic arguments. Pores of these smaller diameters at the outer boundary of the wetted surface will contain water but that water will not participate in the flow (connate water), because of the large capillary force associated with those smaller diameters. Figure 2 suggests that the connate water is contained in pores of 5 microns and less and thus from Figure 9, that capillary effects need not be considered for any pond greater than 90ft. in dimensions. Sullivan (1929), has shown that for ores ground to particle sizes of 74 to 100 microns the effective capillary size was 12 to 14 microns. Since in real dumps the ore fragments are expected to have dimensions of about .1cm (Cathles, 1973) an estimate of 50μ for the effective capillary size of the dump would be a conservative one. Since dump dimensions are greater

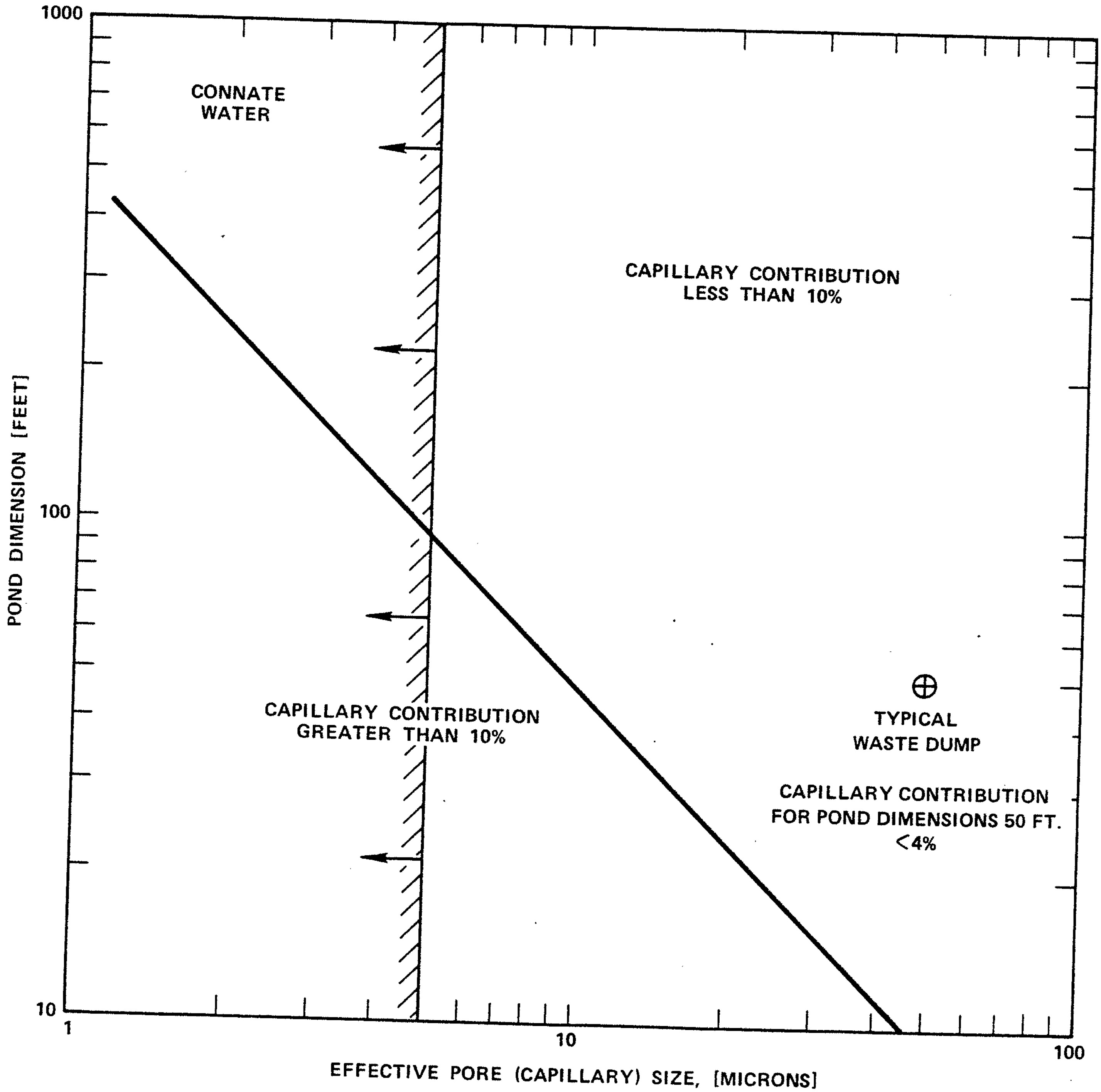
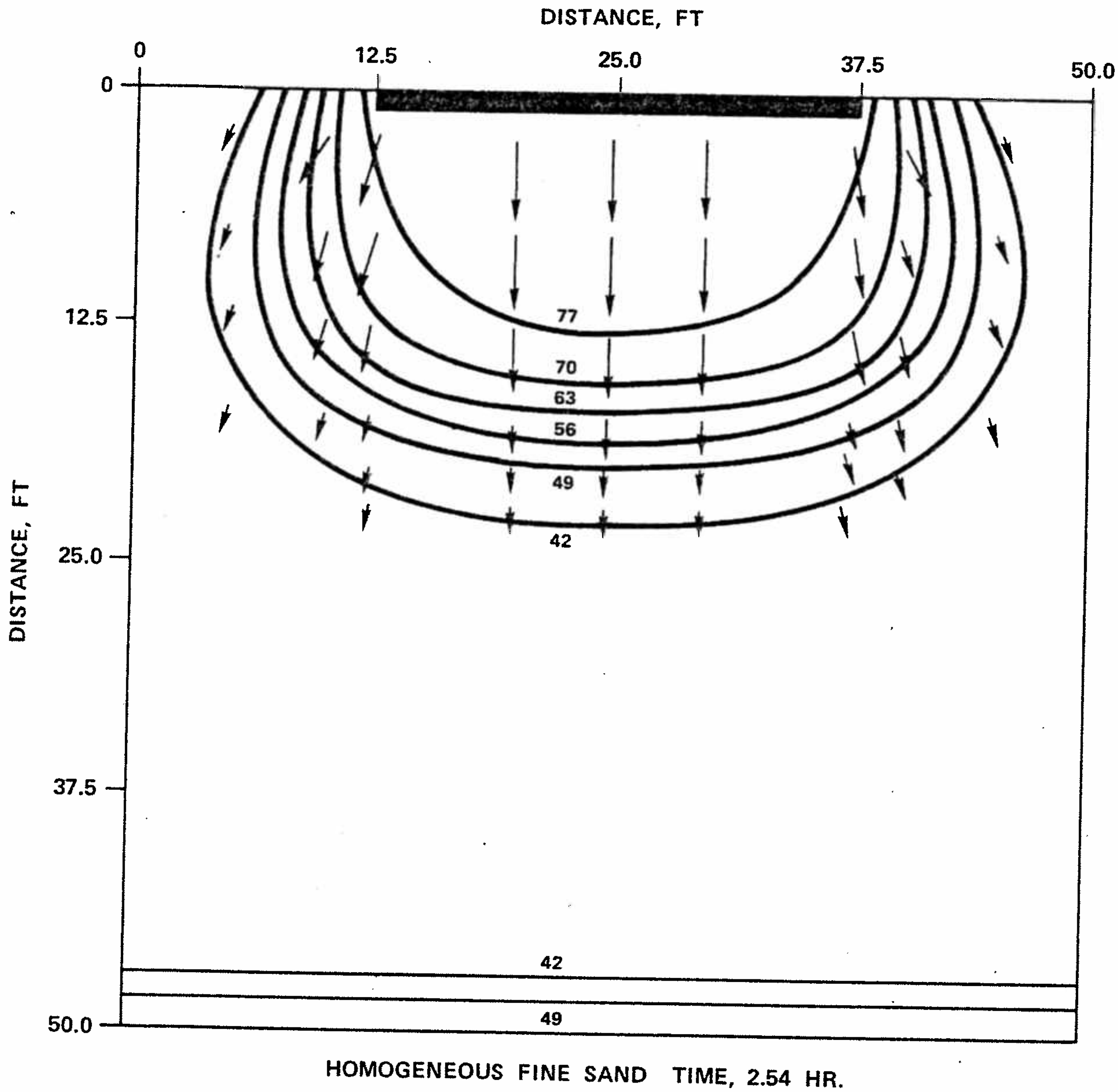


Figure 9. Contribution of the flow in the capillary region to the total flow from a pond into a porous medium.

than 10 feet (Midas test dump, U.C.D. has ponds of 50 feet dimensions) it would appear that capillary effects can be completely neglected when evaluating the flow of the water through the dump, and that the water flow will be essentially vertical within the dump.

The foregoing assumes a homogeneous porous media with a random distribution of pore sizes and that the base of the dump is such that the flow is not confined. Such confinement, i.e. impermeable substrate, could result in a change of the flow pattern, a build up of the water table and increase of the saturation value throughout the dump. Figure 10 shows the results of a computer model analysis of the flow of a shallow pond into a confined area for a time sufficiently short that the confinement has not affected the flow (Jacobson, 1971). Note that Figure 10 is in agreement with the foregoing analyses. Though the author did not specify the pore size distribution in his computer model, the spreading of the flow pattern suggests that h_c is about 6ft and thus the effective pore sizes about 7 microns (a low estimate). The contribution to flow for the capillary zone is nevertheless less than 10%.

In all the analyses discussed both temperature and chemical composition have been assumed to be constant throughout the porous media. Higher temperatures or increased salt content will reduce capillary effects.



EXPLANATION

FLUX VECTOR
1 INCH ≈ 0.02 CM/SEC

50 SATURATION CONTOUR IN PERCENT SATURATION

LEACH POND

Figure 10. Computer model analyses of the flow of a shallow pond into a confined area. The time is sufficiently short that the confinement has not greatly altered the flow pattern (Jacobson, 1971).

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